Modelling Stochastic Volatility of The Stock Market: A Nigerian Experience

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Abstract

In this paper, The GARCH (1,1) model is presented and some results for the existence and uniqueness outlined. Other extensions of the GARCH model including EGARCH, PARCH and TARCH models were presented. The daily stock price of Dangote Cement (Dangocem) was used to test the performance of the above named models with respect to some stylized facts of volatility of financial data: fat tail, volatility clustering, volatility persistence, mean reversion and leverage effect. The Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC) and the Hannan-Quinn criterion (HQ) were used to rate the performance of the models. The results show that the return series are stationary. The summary statistics showed that the return series has a fat tail. From the Q-Q plot, it was seen that the assumption of normality was spurious. The parameter estimation result showed that the volatility of the return series has the mean reversion property. News impact was asymmetric and there is the presence of leverage effect. It was also seen that the volatility process was driven more by negative innovation. Overall the GARCH(1,1) and the TARCH model outperform the other model.

INTRODUCTION:

Globally stock prices are known to be volatile. Investors are interested in getting accurate estimate of the market volatility. This is because volatility is related to profitability of the investment. It is also a known fact that volatility are not directly observable hence the need for models that captures the market volatility. Various models have been developed to capture volatility. The most prominent being the model developed by Engle (1982) 1 the Autoregressive Conditional Heteroskedasticity (ARCH) and generalized by Bollerslev (1986) 2 the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. There are some stylized facts about volatility of financial data readily found in the literatures: Financial data are observed to have excess kurtosis (fat tails) than a Gaussian distribution.

It is also known that period of low volatility tend to follow a period of low volatility while a period of high volatility tends to follow a period of high volatility (volatility clustering). Even though volatility fluctuates highly, it always return to the mean value in the long run (mean reversion).

Volatility has a long memory, that is, it dies down slowly (volatility persistence). It also has been established that bad news increases volatility while good news decreases volatility (leverage effect).

In this paper we seek to model the volatility of a stock listed on the Nigerian Stock Exchange (NSE). The goal is to determine how well the stock price return satisfy some of those stylized facts.

Olowe (2009) 4 used E-GARCH-in-Mean model to investigate the relation between stock returns and volatility in Nigeria. Atoi (2014) 6 Used Nigeria All Share Index to estimates first order symmetric and asymmetric volatility models. The results suggest the presence of leverage effect meaning that volatility responds more to bad news than it does to equal magnitude of good news. Emenike and Aleke (2012) 3 examined the response of volatility to negative and positive news in the Nigerian stock market using daily closing prices of the Nigerian Stock Exchange (NSE) and found asymmetric effects without leverage effect in the NSE stock returns. Hepsag (2016) 11 investigated the asymmetric impact of innovations on volatility and the relationship between the stock return and volatility dynamics in the case of Central and Eastern European (CEE) markets using the framework of asymmetric stochastic volatility models. They found weak evidence of asymmetry, a significant and high volatility persistence in the stock markets of the CEE region. Using EGARCH and TGARCH models, Elsayeda (2011) 5 posited the existence of the leverage effect in the Egyptian stock market index. This they tested with daily EGX30 index returns. Ahmed and Suliman (2011) 7 used different univariate GARCH models to estimate volatility in the daily returns of the Khartoum Stock Exchange (KSE) and found out that the conditional variance process is highly persistent (explosive process). Li (2007) 10 fitted the dynamics of daily stock price data of the Nordea Bank with data from the Nordic Exchange by a class of GARCH models to stock return series. Hojatallah and Ramanarayanan (2010) used the BSE 500 index of Mumbai stock exchange to evaluate the volatility of the Indian stock markets and its related stylized facts using ARCH models. The results suggest that the volatility in the Indian stock market exhibits the persistence of volatility and mean reverting behaviour.

GARCH (1,1) process:

Definition: A random process (ε_t) is called a GARCH (1,1) process if

$$Y_t = X_t^{'}\theta + \epsilon_t \qquad (1)$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (2)$$

With $\omega > 0 \quad \alpha \geq 0,\ \beta \geq 0.$

Equation (1) is the mean equation with an error term while equation (2) is the conditional variance expressed as a function of three terms: ω (a constant), ϵ_{t-1}^2 (the ARCH term) representing news about volatility from the previous period, measured as the lag of the squared residual from the mean equation and σ_{t-1}^2 (the GARCH term) is the last period's forecast variance.

Recursively substituting for the lagged variance on the right-hand side of equation (2), the conditional variance as a weighted average of all the lagged squared residuals

$$\sigma_t^2 = \frac{\omega}{(1-\beta)} + \alpha\sum_{j=1}^{\infty}\beta^{j-1}\epsilon_{t-j}^2 \qquad (3)$$

The error in the squared returns is given by $u_t = \epsilon_t^2 - \sigma_t^2$. Substituting for the variances in the variance equation and rearranging terms we can write our model in terms of the errors

$$\epsilon_t^2 = \omega + (\alpha+\beta)\epsilon_{t-1}^2 + u_t + \beta u_{t-1} \qquad (4)$$

Thus, the squared errors follow a heteroskedastic ARMA (1,1) process. The autoregressive root which governs the persistence of volatility shocks is the sum of $\alpha + \beta$. If the root is very close to unity, the shocks die out slowly.

Conditions for the existence of stationary solution for the GARCH (1,1) process

Definitions:

i. A process is a non-anticipative solution if it is a measurable function of the variable $\varepsilon_{t-s}, S \geq 0$. This implies that σ_t is independent of the σ- field generated by $\{\varepsilon_{t+h}, h > 0\}$.

ii. A stochastic process y_t is stationary if its first and second moments exists and are time invariant that is;

a. E(y_t) = $\mu \; \forall \; t$

b. E$[(y_t - \mu)(y_{t-h} - \mu)'] = \Gamma y(h) = \Gamma y(-h) \; \forall \; t$ and $\mathrm{h} = 0,1,2, \ldots$

Stationarity of the process guarantees that the solution converges and is well defined.

THEOREM 1 18

Given the GARCH (1,1)

$$y_t = \sigma_t \varepsilon_t \tag{5}$$

$$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{6}$$

With $\alpha \geq 0, \alpha \geq 0, \beta \geq 0$.

If we define $a(x) = \alpha x^2 + \beta$ and

$$-\infty \leq g \coloneqq Elog\{\alpha \varepsilon_t^2 + \beta\} < 0. \qquad \text{(6a))}$$

Then the infinite sum

$$h_t = \{1 + \textstyle\sum_{i-1}^{\infty} a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-i})\}\omega \qquad \text{(6b)}$$

converges almost surely $(a.s)$ and the process $(\hat{\varepsilon}_t)$ defined by $(\hat{\varepsilon}_t) = \sqrt{h_t \varepsilon_t}$ is the unique strictly stationary solution of (6)

Proof

For $x > 0$, let $\log_x^+ = max(\log x, 0)$. It is clear that the coefficient $g = E \log[a(\varepsilon_t)] \, \epsilon \, [-\infty, +\infty)$ because $ELog^+\{a(\varepsilon_t)\} \leq Ea(\varepsilon_t) = \alpha + \beta$

Iterating (6)

$$\sigma_t^2 = \alpha_0 + a(\varepsilon_{t-1})\sigma_{t-1}^2$$

$$= \omega[1 + \textstyle\sum_{n-1}^{N} a(\varepsilon_{t-1})\, a(\varepsilon_{t-2}) \cdots a(\varepsilon_{t-n})] + a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-N-1}) \;\; \sigma_{t-N-1}^2$$

$$= h_t(N) + a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-N-1})\sigma_{t-N-1}^2 \tag{7a}$$

Where $N-1 \geq 0$. Since the summands are nonnegative, the limit process

$h_t = \lim_{N\to\infty} h_t(N)$ exist in $\mathbb{R}^+ = [0, \infty)$. Letting N go to infinity in $h_t(N) = \omega + a(\varepsilon_{t-1})h_{t-1}(N-1)$, we get $h_t = \omega + a(\varepsilon_{t-1})h_{t-1}$

We now show that h_t is almost surely finite if and only if $g < 0$.

Suppose $g < 0$ we will get the Cauchy rule for series with nonnegative terms we have ;

$$[a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-n})]^{1/n} = exp\left[\frac{1}{n}\sum_{i=1}^{n} \log\{a(\varepsilon_{t-1})\}\right] \to e^g \text{ a.s.} \quad (7b)$$

As $n \to \infty$, by the application of strong law of large numbers to the iid sequence $(\log\{a(\varepsilon_{t-1})\})$. The series defined in (6b) thus converge almost surely in $\mathbb{R}$, by the application of the Cauchy rule and the limit process (h_t) takes positive real values. It follows that process (y_t) defined by

$$y_t = \sqrt{h_t\varepsilon_t} = \{\alpha_0 + \sum_{i=1}^{\infty} a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-1})\alpha_0\}^{1/2}\eta_t \quad (7c)$$

Is strictly stationary and ergodic. Moreover y_t is a nonanticipative solution of (6).

For uniqueness, we let $-y_t = \sigma_t\varepsilon_t$ be another strictly stationary solution. By (7a) we have

$$\sigma_t^2 = h_t(N) + a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-N-t})\sigma_{t-N-1}^2$$

It follows that;

$$\sigma_t^2 - h_t = \{h_t(N) - h_t\} + a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-N-1})\sigma_{t-N-1}^2$$

The term in brackets on the right-hand side tend to 0 $a \cdot s$ as $N \to \infty$. Moreover, since the series defining h_t converges $a \cdot s$, we have $a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-n}) \to 0$ with probability 1 as $n \to \infty$ in addition, the distribution of σ_{t-N-1}^2 is independent of N by stationarity .Therefore , $a(\varepsilon_{t-1}) \cdots a(\varepsilon_{t-n}) \to 0$ in probability as $N \to \infty$. We have proved that $\sigma_t^2 - h \to 0$ in probability as $N \to \infty$. This term being independent of N, we necessarily have $h_t = \sigma_t^2$ for any t $a \cdot s$.

If $g > 0$, $h_t = +\infty$ $a \cdot s$ it follows that this exists on almost surely finite solution to (6).

For g=0, we give a proof by contradiction suppose there exists a strictly stationary solution (y_t, σ_t^2) of (6) . We have, for $n > 0$.

$$\sigma_0^2 \geq \{1 + \sum_{i=1}^{n} a(\varepsilon_{t-1}) \cdots a(\varepsilon - i)\}$$

From which we deduce that $a(\varepsilon_{-1}) \cdots a(\varepsilon_{-n})\alpha_0$ converges to zero $a \cdot s$, as $n \to \infty$, or equivalently, that

$$\sum_{i=1}^{n} \log a(\varepsilon_i) + \log \alpha_0 \to -\infty \; a \cdot s \text{ as } n \to \infty \quad (7d)$$

By the Chung-Fuchs theorem, we have $\limsup \sum_{i=1}^{n} \log a(\varepsilon_1) = +\infty$ with probability 1, which contradict (7d).■

Proposition 1 (condition for explosion)

For GARCH (1,1) model defined by (1) and (2) given $t \geq 1$, with initial conditions for y_0 and σ_0

$$g > 0 \to \sigma_t^2 \to +\infty \; a \cdot s \;\; (t \to \infty)$$

If in addition, $E|\log(\varepsilon_t^2)| < \infty$ then

$g > 0 \to y_t^2 \to +\infty \; a \cdot s \;\; (t \to \infty).$

Theorem 2 18 **(second order stationarity conditions of the GARCH (1,1) process)**

Let $\omega > 0$. If $\alpha + \beta \geq 1$, a nonanticipative and second order stationary solution to the GARCH (1,1) model does not exist. If $\alpha + \beta < 1$, the process y_t defined by (7c) is second, order stationary. More precisely, y_t is a weak white noise. Moreover, there exists no other second order stationary and non anticipative solution.

Proof: See 18

Distributional Assumptions

There are three assumptions about the conditional distribution of the error term ϵ commonly employed when working with ARCH models: normal (Gaussian) distribution, Student's t-distribution, and the Generalized Error Distribution (GED). Given a distributional assumption, ARCH models are typically estimated by the method of maximum likelihood.

For the GARCH(1, 1) model with conditionally normal errors, the contribution

to the log-likelihood for observation t is:

$$l_t = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log\sigma_t^2 - \frac{1}{2} * \frac{(y_t - X_t')^2}{\sigma_t^2} \qquad (8)$$

where σ_t^2 is specified in one of the forms above.

For the Student's t-distribution, the log-likelihood contributions are of the form:

$$l_t = -\frac{1}{2}\log\left[\frac{\pi(v-2)\Gamma(v/2)^2}{\Gamma\left(\frac{v+1}{2}\right)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \frac{(v+1)}{2}\log\left[1 + \frac{(y_t - X_t'\theta)^2}{\sigma_t^2(v-2)}\right] \qquad (9)$$

where the degree of freedom controls the tail behaviour. The t-distribution approaches the normal as $v \to \infty$

For the GED, we have:

$$l_t = -\frac{1}{2}\log\left[\frac{\Gamma(1/r)^3}{\Gamma(3/r)(r/2)^2}\right] - \frac{1}{2}\log\sigma_t^2 - \left[\frac{\Gamma(3/r)(y_t - X_t'\theta)^2}{\sigma_t^2\Gamma(1/r)}\right]^{r/2}$$

where the tail parameter $r > 0$. The GED is a normal distribution if $r = 2$, and fat-tailed if $r < 2$.

Other GARCH Extensions

EGARCH Model

The EGARCH or Exponential GARCH model was proposed by Nelson (1991). The specification for the conditional variance is:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{q}\beta_j \log\left(\sigma_{t-j}^q\right) + \sum_{i=1}^{p}\alpha_i\left|\frac{\epsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r}\gamma_k\frac{\epsilon_{t-k}}{\sigma_{t-k}} \qquad (10)$$

Recall $\log(\sigma_t^2)$ is the *log* of the conditional variance, hence the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be nonnegative.

PARCH Model

Taylor (1986) and Schwert (1989) introduced the standard deviation GARCH model, where the standard deviation is modelled rather than the variance. The Power ARCH specification follow that. In the Power ARCH model, the power parameter δ of the standard deviation can be estimated rather than imposed, and the optional γ parameters are added to capture asymmetry of up to order:

$$\sigma_t^\delta = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta + \sum_{i=1}^{p} \alpha_i (|\ \epsilon_{t-i}| - \gamma_k \epsilon_{t-i})^\delta \qquad (11)$$

Where $\delta > 0$, $|\gamma_i| \leq 1$ for, $i = 1, \ldots, r$, $\gamma_i = 0$ for all $i > r$, and $r \leq p$.

The symmetric model sets $\gamma_i = 0$ for all i.

We observe that if $\delta = 2$ and $\gamma_i = 0$ for all i, the PARCH model is simply a standard GARCH specification. As in the previous models, the asymmetric effects are present if $\gamma_i \leq 1$

TARCH Model

TARCH or Threshold ARCH and Threshold GARCH sometimes referred to as GJR-GARCH were introduced independently by Zakoïan (1994) and Glosten, Jaganathan, and Runkle (1993). The generalized specification for the conditional variance is given by:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^q + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{r} \gamma_k \epsilon_{t-k}^2 I_{t-k} \qquad (12)$$

where

$$I_t = \begin{cases} 1, & \epsilon_t < 0 \\ 0, & otherwise \end{cases}$$

In this model, good news, $\epsilon_{t-1} > 0$, and bad news. $\epsilon_{t-1} < 0$, have differential effects on the conditional variance; good news has an impact of α_i, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and we say that there is a leverage effect for the i-th order. If $\gamma_i \neq 0$, the news impact is asymmetric.

Data. The data used is Dangote cement (Dangocem) daily stock data from 20th October 2015 to 9th February 2018, a total of 550 observations. The data was gotten from www.cashcraft.com. Fig 1a shows the time series plot of Dangocem price for the time period, while fig 1b is e price the return for the same time period.

Fig 1a Time series of Dangocem daily stock price

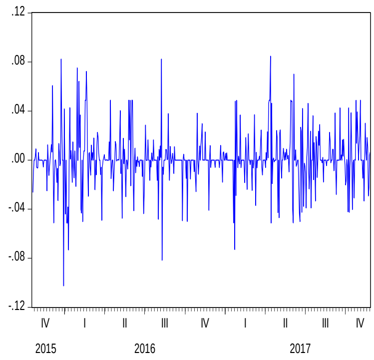

Fig 1b Dangocem return series

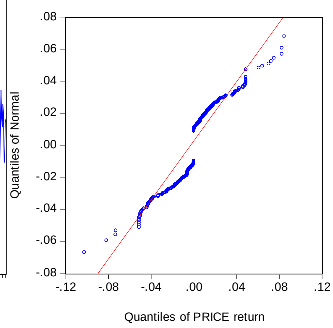

Fig 1c Q-Q plot of the return series

Results

In this work, we estimated the parameters of GARCH (1, 1) , EGARCH, PGARCH AND TARCH of the Dangocem return series and attempt to interpret the result in the light of the stylized facts outlined earlier.

Test for stationarity (Unit root test):

Table (1a) show that Dangote return series is stationary at level since the value of the test statistics -22.11431 is lower than that of the critical values at 1%, 5% and 10%. Thus, the series is stationary of order 0.

Summary Statistics:

The summary statistics (Table 1b) indicates that the return series distribution has a long right tail with skewness of 0.075569 and is peaked (leptokurtic) relative to the normal with a kurtosis of 6.473351(presence of fat tail since $Kurt > 3$). The standard deviation is 0.021653 which translate to a daily variance of 0.0005 or an annualized volatility of 0.36%.

The Q-Q plot

The Q-Q plot (fig 1c) does not lie on the straight line, indicating that the return series is not normally distributed implying that we reject the assumption of normality.

Table 1a: Stationarity test

Table 1b: Summary statistics of stock returns.

Dangocem Stock return			
Mean	0.000832	Skewness	0.075569
Maximum	0.084780	Kurtosis	6.473351
Minimum	-0.10255	Observations	549
Std. Dev.	0.021653		

		AIC	SIC	HQ
ADF Test Statistics		-22.1143	-22.1143	-22.1143
Test critical values:	1% level	-3.44205	-3.44205	-3.44205
	5% level	-2.8666	-2.8666	-2.8666
	10% level	-2.56952	-2.56952	-2.56952

Table 1: Stationarity test

Parameter Estimation

The parameters were estimated for GARCH(1,1), EGARCH, PGARCH and TARCH and three information criteria Akaike, Schwartz and Hann Quinon were utilized to assess the performance of the different models. Recall the model with the least information criterion is said to perform better than the one with greater information criterion. From table 2, TARCH perform better by the AIC and HQ but the GARCH (1,1) perform better by the SIC.

It is observed that except for PARCH model, $\alpha + \beta < 1$, implying that volatility reverts to the mean. The speed of reversion varies; for GARCH (1,1) with $\alpha + \beta = 0.839867413$ it reverts slowly showing persistence while it revert quickly by the EGARCH $\alpha + \beta = 0.44030187$ model.

News impact has asymmetric effect since $\gamma \neq 0$ for all the models. There is also the presence of leverage effect that is bad news increases volatility since $\gamma > 0$ for all the models. From the TARCH model it can be seen that the volatility process is driven more by negative innovation. This is because positive innovation has an impact of -0.02314, while negative innovation has an impact of 0.270927.

The unconditional mean for GARCH(1,1), EGARCH, PGARCH and TARCH are 0.000481 -3.716708349, -0.000289, and 0.000737 respectively. It is obvious that only GARCH(1,1) and TARCH's value that are close to the sample mean of 0.000832 see Table 1b. Thus, we can confirm that GARCH(1,1) and TARCH models perform better as alluded by the Information Criteria.

	GARCH(1,1)	EGARCH	PARCH	TARCH
ω	7.70E-05	-2.08023	0.000461	0.000345
β	0.192087	0.391907	2.417959	0.55542
α	0.64778	0.048395	0.176349	-0.02314
γ		0.772289	0.706957	0.294068
δ			1	
α+β	0.839867	0.440302	2.594308	0.532283
AIC	-5.55121	-5.533	-5.35897	-5.55539
SIC	-5.51982	-5.49377	-5.31189	-5.51616
HQ	-5.53894	-5.51767	-5.34057	-5.54006

Table 2: Parameters estimation for the models

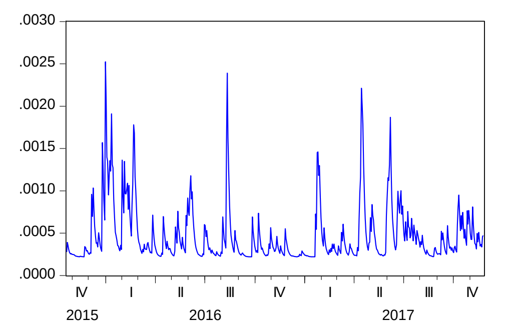

Fig 2: Forecast of variance

Conclusion

In this paper, the GARCH model was presented. In particular, the GARCH (1,1), EGARCH, PGARCH and TARCH were used to model the Dangocem daily stock price from the 20th October 2015 to 9th February 2018, a total of 550 observations. The Unit Root test showed that the return series are stationary also confirmed by the fact that $\alpha + \beta < 1$ for all the model except the PARCH model. The summary statistics showed that the return series has a fat tail with Kurtosis of 6.473351. From the Q-Q plot, it was seen that the return series was not normally distributed hence we could not use the assumption of normality.

The parameter estimation result shows that the volatility of the return series, there is mean reversion since except for the PARCH model $\alpha + \beta < 1$. News impact has asymmetric effect since $\gamma \neq 0$ and there is the presence of leverage effect. Overall the GARCH(1,1) and the TARCH model outperform the other model as was shown by the Information Criteria as well as predicting the forecast variance,

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