SOLVING FRACTIONAL GEOMETRIC PROGRAMMING PROBLEMS VIA RELAXATION APPROACH

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Abstract. In the optimization literature, Geometric Programming problems play a very important role rather than primary in engineering designs. The geometric programming problem is a nonconvex optimization problem that has received the attention of many researchers in the recent decades. Our main focus in this issue is to solve a Fractional Geometric Programming (FGP) problem via linearization technique [1]. Linearizing separately both the numerator and denominator of the fractional geometric programming problem in the objective function, causes the problem to be reduced to a Fractional Linear Programming problem (FLPP) and then the transformed linearized FGP is solved by Charnes and Cooper method which in fact gives a lower bound solution to the problem. To illustrate the accuracy of the final solution in this approach, we will compare our result with the LINGO software solution of the initial FGP problem and we shall see a close solution to the globally optimum. A numerical example is given in the end to illustrate the methodology and efficiency of the proposed approach.

Keywords: Fractional programming, Geometric Programming, Linearization technique.
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1. Introduction

We generally call Fractional Programming , a fractional geometric programming problem(FGPP), if the numerator or denominator or both of them in the objective function as well as the constraints are geometric program. The posynomial fractional programming (PFP) problems are very important and usually arises from the summation minimization of several quotient terms, which are composed of posynomial terms appearing in the objective function. A fractional geometric programming problem(FGPP) is usually denoted as follows:

$$\begin{cases} \min \quad F(x) = \frac{G_0(x)}{G_1(x)} \\ s.t \\ \qquad G_i(x) \leq \delta_i \qquad i = 2, ..., m \\ \qquad x \in \Omega \end{cases}$$

$$\Omega \equiv \left\{ x : 0 \leq \underline{x}_j \leq x_j \leq \bar{x}_j < \infty, j = 1, ..., n \right\}$$

Where

$$G_i(x) = \sum_{t=1}^{T_i} \alpha_{it} \prod_{j=1}^{J_{it}} x_j^{\gamma_{itj}} \qquad \forall i = 0, 1, ..., m.$$

And T_i, J_{it} are subset of natural numbers sets. α_{it} are nonzero real constant coefficients and γ_{itj} are real constant exponents.The geometric programming and fractional programming problems are usually considerd in class of nonlinear programming problems. So far we have seen many different algorithms for solving and analysing GP and FP and in most of them a linear fractional programming problem is being optimized by solving a related linear programming problem. In the recent years we have seen many useful results on solving geometric programming problems by different investigators. Ching-Ter Chang [10] investigated posynomial fractional programming problems In which, he considered the summation minimization of several quotient terms, which are composed of posynomial terms appearing in the objective function and finally a linear programming relaxation was derived for the problem based on piecewise linearization techniques, which first convert a posynomial term into the sum of absolute terms; these absolute terms are then linearized by some linearization techniques. In this paper the proposed approach even after relaxation could reach a solution as close as possible to a global optimum. A global optimization using linear relaxation for generalized geometric programming is recently discussed by Shaojian Qu and et all [11] in which many local optimal solution methods have been developed for solving generalized geometric programming (GGP).They considered the global minimum of (GGP) problems by utilizing an exponential variable transformation and the inherent property of the exponential function and some other techniques that initial nonlinear and nonconvex (GGP) problem is reduced to a sequence of linear programming problems. Geometric programming problems with negative degrees of difficulty are of great concern. S.B. Sinha, et al in [9] they proposed two methods to solve posynomial geometric programs with negative degrees of difficulty. We deal with Such a case ,when a primal program has a number of variables equal or slightly greater than the number of terms appear in the model. Problem of Posynomial parametric geometric programming with interval valued coefficient is discussed by Mahapatra and Mandal [12] in which they proposed a parametric functional form of an interval number by solving directly the objective function of geometric programming without transferring to dual form. Zahmatkesh and Cao[17] proposed a method to solve global optimization of fractional posynomial geometric programming problems under fuzziness in which they used trapezoidal fuzzy number in the objective function.The problem of posynomial geometric programming with parametric uncertainty is discussed by Shiang-Tai Liu [13]. He developed a procedure to derive the lower and upper bounds of the objective of the posynomial geometric programming problem when the cost and constraint parameters are uncertain and the imprecise parameters were represented by ranges, instead of single values and finally an imprecise geometric program was transformed to a family of conventional geometric programs to calculate the objective value. The problem of a profit maximization is considerd by Li,Yiming,Ying-Chien Chen [4]. When quantity discount is involved in profit maximization is the work of Liu, Shiang-Tai [5]. Multi-objective marketing planning inventory model is discussed by Islam and Sahidul [3]. Wu,Yan-Kuen discussed the problem of optimizing a geometric programming with single-term exponent subject to max-min fuzzy relational equation constraints [2]. A good work on linearization is done by Shajian Qu and etal [6] in which they considered the problem of a global optimization using linear relaxation for generalized geometric programming. A very recent research on geometric programming problems is done by Saraj and Bazikar [8] in which they have investigated the problem of linear multi-objective geometric programming problem via reference point approach. The problem of multi objective geometric programming problem with ϵ-constraint method is a recent work done by Ojha and Biswal [14] in which they have used ϵ-constraint method to find the non-linear solution of the multi-objective programming problems. An interesting paper in generalized geometric programming problem is due to Jung-Fa Tsi and et.al [15]. In their paper they have proposed a technique for treating non-positive variables with integer powers in general geometric programming problems by means of variable transformation. In 2010,

A.K Ojha and A.K Das [16] investigated the problem of multi objective geometric programming with cost coefficient as continuous function and solved with mean method.
The present article is organized as follows. In section 2, we construct a linear relaxation to obtain a lower bound function[1] for the objective and constraint functions to transform the FGP to a LFP. In section 3, we use Charnes and Cooper method [7] to solve the transformed linear fractional programming problem. A numerical example is provided in section 4 to implement and clear the complexity of the analysis for our proposed approach. Finally a brief conclusion is given in the end.

2. LINEAR RELAXATION TECHNIQUE

The proposed strategy for generating linear programming relaxation is to underestimate every nonlinear function $G_i(x)(i = 0, 1, ...m)$. Let $\Omega_j \equiv \{x_j : \underline{x}_j \leq x_j \leq \overline{x}_j\}$, we use $L_{itj}^{\Omega_j}(x_j)$, $U_{itj}^{\Omega_j}(x_j)$ to denote the nonnegative lower and upper bound of linear approximate functions $x_j^{\gamma_{itj}}$ over Ω_j for $j \in J_{it}$. $l_{itj}^{\Omega_j}(x_j)$ is to denote the straight line through points $(\underline{x}_j, \underline{x}_j^{\gamma_{itj}})$ and $(\overline{x}_j, \overline{x}_j^{\gamma_{itj}})$, $l_{itj}^{*\Omega_j}(x_j)$ to denote the straight line that is tangent to $x^{\gamma_{itj}}$ at $(x_{itj}^*, {x_{itj}^*}^{\gamma_{itj}})$ where $x_{itj}^* = \left[\frac{\gamma_{itj}\left(\bar{x}_j - \underline{x}_j\right)}{\bar{x}_{itj}^{\gamma_{itj}} - \underline{x}_{itj}^{\gamma_{itj}}}\right]^{\frac{1}{\left(1-\gamma_{itj}\right)}}$
, $as \quad \gamma_{itj} \neq 1$.

$$l_{itj}^{\Omega_j}(x_j) = \underline{x}_j^{\gamma_{itj}} + \frac{\bar{x}_j^{\gamma_{itj}} - \underline{x}_j^{\gamma_{itj}}}{\bar{x}_j - \underline{x}_j}\left(x_j - \underline{x}_j\right)$$

$$l_{itj}^{*\Omega_j}(x_j) = {x_{itj}^*}^{\gamma_{itj}} + \frac{\bar{x}_j^{\gamma_{itj}} - \underline{x}_j^{\gamma_{itj}}}{\bar{x}_j - \underline{x}_j}\left(x_j - x_{itj}^*\right)$$

We need to introduce the following two straigth lines $\tilde{l}_{itj}^{\Omega_j}(x_j)$ and $\hat{l}_{itj}^{\Omega_j}(x_j)$,where $\tilde{l}_{itj}^{\Omega_j}(x_j)$ passes through the point $(\underline{x}_j, 0)$ and is tangent to $x_j^{\gamma_{itj}}$ at the point $\left(\tilde{x}_{itj}, \tilde{x}_{itj}^{\gamma_{itj}}\right)$,and $\hat{l}_{itj}^{\Omega_j}(x_j)$ passes through the point $(\overline{x}_j, 0)$ and is tangent to $x_j^{\gamma_{itj}}$ at the point $\left(\hat{x}_{itj}, \hat{x}_{itj}^{\gamma_{itj}}\right)$, where:

$$\tilde{x}_{itj} = \left(\frac{\gamma_{itj}}{\gamma_{itj} - 1} \right) \underline{x}_j, \ \ (as \quad \gamma_{itj} > 1 \quad)$$

$$\tilde{l}_{itj}^{\Omega_j}(x_j) = \gamma_{itj} \tilde{x}_{itj}^{\gamma_{itj}-1} \left(x_j - \underline{x}_j \right), \quad as \quad \gamma_{itj} > 1.$$

and

$$\hat{x}_{itj} = \left(\frac{\gamma_{itj}}{\gamma_{itj} - 1} \right) \bar{x}_j, \ \ (as \quad \gamma_{itj} < 0 \quad)$$

$$\hat{l}_{itj}^{\Omega_j}(x_j) = \gamma_{itj} \hat{x}_{itj}^{\gamma_{itj}-1} \left(x_j - \bar{x}_j \right), \quad as \quad \gamma_{itj} < 0.$$

2.1.First-stage relaxation :

If $\alpha_{it} > 0$, then

$$L_{itj}^{\Omega_j}(x_j) = \begin{cases} l_{itj}^{*\Omega_j}(x_j) & when \quad \gamma_{itj} > 1 \quad \& \ \ l_{itj}^{*\Omega_j}\left(\underline{x}_j\right) \geq 0 \\ \tilde{l}_{itj}^{\Omega_j}(x_j) & when \quad \gamma_{itj} > 1 \quad \& \ \ l_{itj}^{*\Omega_j}\left(\underline{x}_j\right) < 0 \\ x_j & when \quad \gamma_{itj} = 1 \\ l_{itj}^{\Omega_j}(x_j) & when \quad 0 < \gamma_{itj} < 1 \\ l_{itj}^{*\Omega_j}(x_j) & when \quad \gamma_{itj} < 0 \quad \& \ \ l_{itj}^{*\Omega_j}(\bar{x}_j) \geq 0 \\ \hat{l}_{itj}^{\Omega_j}(x_j) & when \quad \gamma_{itj} < 0 \quad \& \ \ l_{itj}^{*\Omega_j}(\bar{x}_j) < 0 \end{cases}$$

If $\alpha_{it} < 0$, then

$$U_{itj}^{\Omega_j}(x_j) = \begin{cases} l_{itj}^{\Omega_j}(x_j) & when \quad \gamma_{itj} > 1 \quad or \quad \gamma_{itj} < 0 \\ l_{itj}^{*\Omega_j}(x_j) & when \quad 0 < \gamma_{itj} < 1 \\ x_j & when \quad \gamma_{itj} = 1 \end{cases}$$

Therefore we can get a lower bounded function for the $\alpha_{it} \prod_{j=1}^{J_{it}} x_j^{\gamma_{itj}}$ as

$$\alpha_{it} \prod_{j=1}^{J_{it}} x_j^{\gamma_{itj}} \geq G_{it}^{R(\Omega)}(x)$$

when

$$G_{it}^{R(\Omega)}(x) \equiv \begin{cases} \alpha_{it} \prod_{j=1}^{J_{it}} L_{itj}^{\Omega_j}(x_j) & if \quad \alpha_{it} > 0 \\ \alpha_{it} \prod_{j=1}^{J_{it}} U_{itj}^{\Omega_j}(x_j) & if \quad \alpha_{it} < 0 \end{cases} \qquad \forall i, t.$$

Over all we have the terms $t \in T_i$ for each $i = 0, 1, ..., m$, $\sum_{t=1}^{T_i} G_{it}^{R(\Omega)}(x)$ as $G_i^{R(\Omega)}(x)$ and $G_i(x) \geq G_i^{R(\Omega)}(x)$, $\forall x \in \Omega$, $i = 0, 1, ...m$ .Therefore the first linear relaxation programming problem (FLR) can be obtained as follows

$$FLR(\Omega) : \begin{cases} \min & \frac{G_0^{R(\Omega)}(x)}{G_1^{R(\Omega)}(x)} \\ s.t & \\ & G_i^{R(\Omega)}(x) \leq \delta_i \quad i = 2, ..., m \\ & x \in \Omega \end{cases}$$

2.2.Second-stage relaxation :

Theorm1. The function $l(y) = \prod_{j=1}^{p} y_j$ has lower and upper bounded linear functions $q_{11}(y)$, $q_{12}(y)$ and $q_{21}(y)$, $q_{22}(y)$ over $\overline{R} = \{y \in R^p :$

$\underline{\beta}_j \leq y_j \leq \overline{\beta}_j, j = 1, 2, ..., p\}$.

$$\begin{cases} q_{11}(y) = \sum_{j=1}^{p} \left(\prod_{\substack{i=1 \\ i \neq j}}^{p} \underline{\beta}_i \right) y_j - (p-1) \prod_{j=1}^{p} \underline{\beta}_j \\ q_{12}(y) = \sum_{j=1}^{p} \left(\prod_{\substack{i=1 \\ i \neq j}}^{p} \bar{\beta}_i \right) y_j - (p-1) \prod_{j=1}^{p} \bar{\beta}_j \\ q_{21}(y) = \sum_{j=1}^{p} \left(\prod_{i=1}^{j-1} \bar{\beta}_i \right) \left(\prod_{i=j+1}^{p} \underline{\beta}_i \right) y_j - \sum_{j=1}^{p-1} \left(\prod_{i=1}^{j} \bar{\beta}_i \right) \left(\prod_{i=j+1}^{p} \underline{\beta}_i \right) \\ q_{22}(y) = \sum_{j=1}^{p} \left(\prod_{i=1}^{j-1} \underline{\beta}_i \right) \left(\prod_{i=j+1}^{p} \bar{\beta}_i \right) y_j - \sum_{j=1}^{p-1} \left(\prod_{i=1}^{j} \underline{\beta}_i \right) \left(\prod_{i=j+1}^{p} \bar{\beta}_i \right) \end{cases}$$

and $l(y) = q_{11}(y)$ for all $y \in \{\underline{\beta}\} \cup N_1$ where N_1 denotes the set of all extreme points of $\overline{R}$ adjacent to $\underline{\beta}$, $l(y) = q_{12}(y)$ for all $y \in \{\overline{\beta}\} \cup N_2$ where N_2 denotes the set of all extreme points of $\overline{R}$ adjacent to $\overline{\beta}$.

For proof ,see Ref [1].

Now ,if $L_{itj}^{\Omega_j}(x_j)$ is increasing over $[\ \underline{x}_j\ ,\ \bar{x}_j\ ]$, then let $\underline{L}_{itj} = L_{itj}^{\Omega_j}\left(\underline{x}_j\right)$,$\bar{L}_{itj} = L_{itj}^{\Omega_j}(\bar{x}_j)$ otherwise ,let $\underline{L}_{itj} = L_{itj}^{\Omega_j}(\bar{x}_j)$, $\bar{L}_{itj} = L_{itj}^{\Omega_j}\left(\underline{x}_j\right)$ .If $U_{itj}^{\Omega_j}(x_j)$ is increasing over $[\ \underline{x}_j\ ,\ \bar{x}_j\ ]$,then let $\underline{U}_{itj} = U_{itj}^{\Omega_j}\left(\underline{x}_j\right)$,$\bar{U}_{itj} = U_{itj}^{\Omega_j}(\bar{x}_j)$,otherwise ,let $\underline{U}_{itj} = U_{itj}^{\Omega_j}(\bar{x}_j)$, $\bar{U}_{itj} = U_{itj}^{\Omega_j}\left(\underline{x}_j\right)$. Then

$$G_{it}^{R(\Omega)}(x) \geq \bar{G}_{it}^{R(\Omega)}(x) \quad , \quad \forall x \in \Omega \quad , \quad i = 0, 1, ...m.$$

When $\mid J_{it} \mid > 1$,

$$\bar{G}_{it}^{R(\Omega)}(x) = \begin{cases} \alpha_{it}\left(\sum_{j=1}^{J_{it}}\left(\prod_{\substack{k=1\\k\neq j}}^{J_{it}}\underline{L}_{itk}\right)L_{itj}^{\Omega_j}(x_j) - (|J_{it}| - 1)\prod_{k=1}^{J_{it}}\underline{L}_{itk}\right) & if \quad \alpha_{it} > 0 \\ \alpha_{it}\left(\sum_{j=1}^{J_{it}}\left(\prod_{\substack{k=1\\k<j}}^{J_{it}}\bar{U}_{itk}\right)\left(\prod_{\substack{k=1\\k>j}}^{J_{it}}\underline{U}_{itk}\right)U_{itj}^{\Omega_j}(x_j)\right. & \\ \left. - \sum_{j=1}^{J_{it}-\{\max J_{it}\}}\left(\prod_{\substack{k=1\\k\leq j}}^{J_{it}}\bar{U}_{itk}\right)\left(\prod_{\substack{k=1\\k>j}}^{J_{it}}\underline{U}_{itk}\right)\right) & if \quad \alpha_{it} < 0 \end{cases}$$

and when $\mid J_{it} \mid = 1$,

$$\bar{G}_{it}^{R(\Omega)}(x) = \begin{cases} L_{itj}^{\Omega_j}(x_j) & if \quad \alpha_{it} > 0 \\ U_{itj}^{\Omega_j}(x_j) & if \quad \alpha_{it} < 0 \end{cases} \qquad \forall i, t.$$

and

$$\tilde{G}_{i}^{R(\Omega)}(x) = \sum_{t=1}^{T_i}\bar{G}_{it}^{R(\Omega)}(x) \ , \quad \forall t \in T_i, j \in J_{it} \quad , \quad i = 0, 1, ..m.$$

Thus the second linear relaxation programming problem (SLR) over Ω can be described as follows:

$$SLR(\Omega) : \begin{cases} \min \quad \tilde{F}^{R(\Omega)} = \frac{\tilde{G}_0^{R(\Omega)}(x)}{\tilde{G}_1^{R(\Omega)}(x)} \\ s.t \\ \qquad \tilde{G}_i^{R(\Omega)}(x) \leq \delta_i \qquad i = 2, ..., m \\ \qquad x \in \Omega \end{cases}$$

For demonstrating the behaviour of the optimal objective function value, the following relation is always true.

V[SLR(Ω)] $\leq$ V[FLR(Ω)] $\leq$ V[FGP(Ω)].

Where V[MOGP(Ω)] stands for the primal objective function value , V[FLR(Ω)] the value of objective function in the first-stage of linear relaxation, where as V[SLR(Ω)] represents the value of objective function in the second-stage of the linear relaxation problem. As we know the main struture in the development of a solution procedure for solving the problem of (FGP) is the construction of lower bounds for this problem.Therefore a linear programming relaxation problem can be solved to obtain a lower bound for the solution of problem.

3. Generalize problem

Consider the following FGP

$$\begin{cases} \min \quad F(x) = \frac{G_0(x)}{G_1(x)} \\ s.t \\ \qquad G_i(x) \leq 1 \qquad i = 2, .., m \end{cases}$$

Where

$$G_i(x) = \sum_{t=1}^{T_i} \alpha_{it} \prod_{j=1}^{T_{it}} x_j^{\gamma_{itj}} \quad , \quad i = 0, 1, .., m$$

then

$$FLR(\Omega) : \begin{cases} \min \quad F^{R(x)}(x) = \frac{G_0^{R(x)}(x)}{G_1^{R(x)}(x)} = \frac{\sum_{t=1}^{T_0} G_{0t}^{R(x)}(x)}{\sum_{t=1}^{T_1} G_{1t}^{R(x)}(x)} \\ s.t \\ \qquad G_i^{R(x)}(x) = \sum_{t=1}^{T_i} G_{it}^{R(x)}(x) \leq 1 \qquad , i = 2, ..., m \\ \qquad x \in \Omega \end{cases}$$

where

$$G_i^{R(\Omega)}(x) = \sum_{t=1}^{T_i} G_{it}^{R(\Omega)}(x) = \underbrace{\sum_{t=1}^{T_i} \alpha_{it} \prod_{j=1}^{J_{it}} L_{itj}^{\Omega_j}(x_j)}_{\alpha_{it}>0} + \underbrace{\sum_{t=1}^{T_i} \alpha_{it} \prod_{j=1}^{J_{it}} U_{itj}^{\Omega_j}(x_j)}_{\alpha_{it}<0} \quad , i = 0, 1, ..., m$$

and

$$SLR(\Omega) : \begin{cases} \min \quad \tilde{F}^{R(\Omega)}(x) = \frac{\tilde{G}_0^{R(\Omega)}(x)}{\tilde{G}_1^{R(\Omega)}(x)} = \frac{\sum_{t=1}^{T_0} \bar{G}_{0t}^{R(\Omega)}(x)}{\sum_{t=1}^{T_1} \bar{G}_{0t}^{R(\Omega)}(x)} \\ s.t \\ \qquad \tilde{G}_i^{R(\Omega)}(x) = \sum_{t=1}^{T_i} \bar{G}_{it}^{R(\Omega)}(x) \le 1 \qquad , i = 2, ..., m \\ \qquad x \in \Omega \end{cases}$$

where

$$\sum_{t=1}^{T_i} \bar{G}_{it}^{R(\Omega)}(x) = \underbrace{\sum_{t=1}^{T_i} \alpha_{it} \left[\left(\sum_{j=1}^{J_{it}} \frac{\left(\prod_{\substack{k=1 \\ k \neq j}}^{J_{it}} itk \right) L_{itj}^{\Omega_j}(x_j) -}{(|J_{it}| - 1) \prod_{k=1}^{J_{it}} itk} \right) + L_{itj}^{\Omega_j}(x_j) \right]}_{\alpha_{it}>0}$$

$$+ \underbrace{\sum_{t=1}^{T_i} \alpha_{it} \left[\left(\sum_{j=1}^{J_{it}} \frac{\left(\prod_{\substack{k=1 \\ k<j}}^{J_{it}} \bar{U}_{itk} \right) \left(\prod_{\substack{k=1 \\ k>j}}^{J_{it}} itk \right) U_{itj}^{\Omega_j}(x_j) -}{\sum_{j=1}^{J_{it}-\{\max J_{it}\}} \left(\prod_{\substack{k=1 \\ k \le j}}^{J_{it}} \bar{U}_{itk} \right) \left(\prod_{\substack{k=1 \\ k>j}}^{J_{it}} itk \right)} \right) + U_{itj}^{\Omega_j}(x_j) \right]}_{\alpha_{it}<0}$$

$, i = 0, 1, .., m$

3.1 Charnes and Cooper method for LFP

One of the earliest method to solve LFP was developed by Charnes and Cooper in[7]. In this method a LFP is converted into a LP by proper change of variables. The denominator of the fractional objective function $F(x)$ is taken equal to which by assumption is not equal to zero and it is also further assumed that it is positive everywhere. If it is negative then the negative sign is taken to the numerator. Now the LFP is converted into LP with variables y_j and ν by using the following substitutions:

$$\frac{x_j}{\nu} = y_j \geq 0 \ , \quad 1 \leq j \leq n$$

$$\frac{1}{\nu} = u \geq 0$$

The substitution $a_0 + a_1x_1 + ... + a_nx_n = \nu \neq 0$ is taken as a constraint and it is given as

$$a_1y_1 + ... + a_ny_n + a_0u = 1$$

Then the equivalent LP matrix formation of given LFP is

$$\begin{cases} \min \quad z = c_0u + c_1y_1 + ... + c_ny_n \\ s.t \\ \qquad D_{n\times n}y_{n\times 1} - d_{n\times 1}u \leq 0 \\ \qquad a_0u + a_1y_1 + ... + a_ny_n = 1 \end{cases}$$

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & d_{nn} \end{bmatrix} \ , \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad , \quad d = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}$$

The most advantage of the proposed method as we see is to make solvable a fractional geometric programming in which both the numerator and denominator of the objective function is geometric program.

4.Numerical expriment

Consider the following example

$$\begin{cases} \min \quad F(x) = \frac{-x_1 + x_1 x_2^{0.5} - x_2}{4x_1^2 x_2^{-3} + 5x_1^{-3} x_2 + 7x_1 x_2} \\ s.t \\ \qquad -6x_1 + 8x_2 \leq 3 \\ \qquad 3x_1 - x_2 \leq 3 \\ \qquad 0.1 \leq x_1 \leq 1.5 \\ \qquad 0.1 \leq x_2 \leq 1.5 \end{cases}$$

Then by using linear relaxation technique we obtain the second-stage approximating to the problem as:

$$\begin{cases} \min \quad \tilde{F}^{R(\Omega)}(x) = \frac{-0.68x_1 - 0.94x_2 - 0.01}{-0.24x_1 + 0.7x_2 + 1.33} \\ s.t \\ \qquad -6x_1 + 8x_2 \leq 3 \\ \qquad 3x_1 - x_2 \leq 3 \\ \qquad 0.1 \leq x_1 \leq 1.5 \\ \qquad 0.1 \leq x_2 \leq 1.5 \end{cases}$$

On using Charnes and Cooper technique, the above LFPP is reduced to the following LPP.

$$\begin{array}{ll} \min & z = (-0.68)y_1 + (-0.94)y_2 + (-0.01)u \\ s.t & \\ & -6y_1 + 8y_2 - 3u \leq 0 \\ & 3y_1 - y_2 - 3u \leq 0 \\ & (-0.24)y_1 + (0.7)y_2 + (1.33)u = 1 \\ & y_1, y_2, u \geq 0 \end{array}$$

Where the solution of above LP is give by

$$\begin{cases} y_1 = 0.74 \\ y_2 = 0.74 \\ u = 0.49 \\ x_1 = 1.5 \\ x_2 = 1.5 \\ z^* = -1.21 \end{cases} \quad \cdots \quad (1)$$

Now we shall solve the primary FGPP by LINGO software, and we get

$$\begin{cases} x_1 = 1.1 \\ x_2 = 1.2 \\ z^* = -0.066 \end{cases} \qquad \cdots \quad (2)$$

As we see, there is an acceptable convergance regarding the values x_1 and x_2 while we compare the set of solutions in (1) and (2).

5.CONCLUSION

As we know geometric programming and fractional geometric programming are entirely two different issues in sense of method of solutions which are hardly non-convex problems. In the present work we have combined both the problems and have introduced a new problem called Fractional geometric programming problems (FGPP). In fact such problems are not easy to solve as such. By employing relaxation technique [1] on both geometric programming of the numerator and denominator of the FGPP, we have made solvable and easy the solution of the problem by converting into a linear fractional programming which can be easily solved by Charnes and Cooper technique [7]. It is obvious that the linearization technique. Which is used here is much more convenient in the sense of computationally compare to convex relaxation, since many conditions must be taken into account while doing convexification.

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