Determination of the JS- Maximal soluble subgroups of the General linear Group in Dimentions 14,15 and 16 Over a Filed of p^k Elements.

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Abstract

In this paper we will compute all of the JS- maximal soluble subgroups of the groups $GL(14, p^k)$, $GL(15, p^k)$ and $GL(16, p^k)$. It turns out the number of types of the JS- maximal soluble subgroups in the groups $GL(14, p^k)$, $GL(15, p^k)$ and $GL(16, p^k)$ are 14,10 and 47 respectively. Furthermore we find the structure of these subgroups.

Key words: General linear Group, Grthogonal Group, primitive Group, symplectic Group..
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Introduction

In [7] C. Jordan determined a table containing the number of conjugacy classes of maximal irreducible soluble subgroups of the group $GL(n, p)$, for $p^n < 106$. And In [6] Iĺin and Takamakov determamined the primitive simple permutation groups of small degree. And also In [13] Martin determined the primitive substitution groups of degree fifteen and the primitive substitution groups of degree eighteen. $L.E$. Dickson in [3] determined all subgroups of $PSP(2, p^k)$ and also in [4] he determined all subgroups of $PSP(4, 3)$.$E.R$ Bennett in [1] computed the primitive groups of degree 20. Howard H.Mitchell in [14] determined the maximal subgroups of $PSP(4, p^k)$ for odd p. S.G. Liskovecl in [12] classified the maximal irreducible (p, q)- subgroups of $GL(r^2, p)$, where q and r are primes and q is odd. In [2], $S.B$. Conlon determined the non - abelian q subgroups (q prime) of the group $GL(q, p^n)$ and the non-abelian 2-subgroups of $SP(2, p^k)$. In [5] K. Harada and H. Yamaki found the irreducible subgroups of the group $GL(n, 2)$ for $n \leq 6$. And In [8], [9], [10] and [11], $A.S$ Kondrat'ev determined the irreducible subgroups of the group $GL(7, 2)$, the insoluble irreducible subgroups of the groups $GL(8, 2)$ and $GL(9, 2)$ and the insoluble primitive subgroups of $GL(10, 2)$, respectively.

In the early 1960's. Sims developed an algorithm, based on coset enumeration, which takes as input a group G given by a finite representation and positive integer n, and output a list containing representatives of each conjugacy class of subgroups of G whose index is at most n. A similar algorithm was developed independently by Schaps in [15].$L.G$. Kovacs, J.Nübuser and $M.F$. Newman (unpublished notes) have proposed an algorithm which computes certain maximal subgroups of low index.

In [16], $M.W.$ Short determined the primitive soluble permutation groups of degree less than 256 and in [15] B. Razzaghmaneshi Havigh determined the JS- maximal soluble subgroups of the general linear group in dimensions 8, 9, 10 and 12. Now in this paper we will determine the JS- maximal soluble subgroups of the groups $GL(14, p^k), GL(15, p^k)$ and $GL(16, p^k)$.

We use of the methods of [17] and it turns out that the groups $GL(14, p^k)$, $GL(15, p^k)$ and $GL(15, p^k)$ have $14, 10$ and 27 JS- maximal soluble subgroup respectively. The term JS- maximal is used for Jordan and Suprunenko subgroups of the group $GL(n, F)$. By [17] any group constructed by Theorems 2.5.9, 2.5.35 and 2.5.37, is called a JS- maximal soluble subgroup of the group $GL(n, F)$. The terms JS - imprimitive and JS -primitive are used for imprimitive and primitive JS- maximal soluble subgroups respectively.

Definition: (i) If G, N and H are groups and G has a normal subgroup N_0 isomorphic to N such that GN_0 is isomorphic to H, then we write $G = NH$.

(ii) If G has a subgroup isomorphic to H which intersects N_0 trivially. Then G is a semidirect product of N and H, and we write $G = NH$.

(iii) The holomorph of a group G, written Hol(G), is the semidirect product of G and its automorphism group.

Notation: throughout this paper we use Sym (X) to denote the symmetric group on the set X, and S_n to mean the symmetric group on the set of the first n positive integers, if G and H are permutation groups, we denote the wreath product of G and H by G wr H, where G is a coordinate subgroup and H is the top group. And also if $H_1, \ldots, H_n$ are n groups, then we denote the central product of H_1, H_2, $\ldots$, H_n by $H_1 H_2 \ldots H_n$.

Main Theorems

Theorem 3. Let p be a prime number and k, n be positvive integers and let F be the field of p^k elements. Then the number of types of JS- maximal soluble subgroups in the group $GL(14, p^k)$ is 14.

Theorem 6. Let p be a prime number and k, n be positive integers and let F be the field of p^k elements. Then the number types of JS- maximal soluble subgroups of the group $GL(15, p^k)$ is 10.

Lemma 8. Let p be a prime number and k, n be positive integers and let F be the field of p^k elements. Then the number of types of JS-primitive maximal soluble subgroups in the group $GL(16, p^k)$ is 27.

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