n$\mathcal{I}_{\ddot{g}}$-Closed Sets

[1]S. Ganesan, [2]C. Alexander [3]S. Jeyashri and [4]S. M. Sandhya

[1,2]Assistant Professor, PG & Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.
(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)
[3]Assistant Professor, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal-624102, Tamil Nadu, India.
[4] Ph.D Scholar, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal-624102, Tamil Nadu, India.
[1]sgsgsgsgsg77@gmail.com, [2]alexchinna07@yahoo.com, [3]balasri127@gmail.com, [4]sandhyasmtvm@gmail.com (Received: February/2020, Accepted: April/2020)

Abstract
Characterizations and properties of n$\mathcal{I}_{\ddot{g}}$-closed sets and n$\mathcal{I}_{\ddot{g}}$-open sets are given. The main purpose of this paper is to introduce the concepts of sg-n$\mathcal{I}$-locally closed sets, n$\wedge_{sg}$-sets, η_{sg}-n$\mathcal{I}$-closed sets, n$\mathcal{I}_{\ddot{g}}$-continuous, sg-n$\mathcal{I}$-LC-continuous, η_{sg}-n$\mathcal{I}$-continuous and to obtain decompositions of n$\star$-continuity in nano ideal topological spaces.

Keywords: n$\mathcal{I}_{\ddot{g}}$-closed sets, n$\mathcal{I}_{\ddot{g}}$-open sets, n$\mathcal{I}_{\ddot{g}}$-continuous, sg-n$\mathcal{I}$-LC-continuous and η_{sg}-n$\mathcal{I}$-continuous.

1 Introduction

Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on U, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n : \wp(U) \to \wp(U)$ ($\wp$(U) is the set of all subsets of U) [8, 9]. For a subset A $\subseteq$U, $A^*_n(\mathcal{I}, \mathcal{N})=\{x \in U : G_n \cap A \notin \mathcal{I}$, for every $G_n \in G_n(x)\}$, where $G_n=\{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$ is called the nano local function(briefly n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for $A^*_n(\mathcal{I}, \mathcal{N})$. Parimala et al [9] introduced the concept of nano ideal topological spaces and investigated some of its basic properties.

In this paper, we introduce a new class of sets namely n$\mathcal{I}_{\ddot{g}}$-closed sets in nano ideal topological spaces. This class lie between the class of n$\star$-closed sets and the class of n$\mathcal{I}_g$-closed sets. Characterizations and properties of n$\mathcal{I}_{\ddot{g}}$-closed sets and n$\mathcal{I}_{\ddot{g}}$-open sets are studied. Finally, we obtain decompositions of n$\star$-continuity in nano ideal topological spaces.

Moreover the study of n$\mathcal{I}_{\ddot{g}}$-closed sets led to some n$T_{\mathcal{I}}$-space are extensively developed and used in computer science and digital topology.

2 Preliminaries

Theorem 2.1. *[8, 9] Let (U, $\mathcal{N}$) be a nano topological space with ideal $\mathcal{I}$, $\mathcal{I}'$ on U and A, B be subsets of U. Then*

1. $A \subseteq B \Rightarrow A^*_n \subseteq B^*_n$.
2. $\mathcal{I} \subseteq \mathcal{I}' \Rightarrow A^*_n(\mathcal{I}') \subseteq A^*_n(\mathcal{I})$.

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3. A^*_n=n-cl(A^*_n) $\subseteq$ n-cl(A) (A^*_n is a nano closed subset of n-cl(A)).
4. $(A^*_n)^*_n \subseteq A^*_n$.
5. $A^*_n \cup B^*_n = (A \cup B)^*_n$
6. $A^*_n - B^*_n = (A - B)^*_n - B^*_n \subseteq (A - B)^*_n$.
7. $V \in \mathcal{N} \Rightarrow V \cap A^*_n = V \cap (V \cap A)^*_n \subseteq (V \cap A)^*_n$ and
8. $J \in \mathcal{I} \Rightarrow (A \cup J)^*_n = A^*_n = (A - J)^*_n$

Lemma 2.2. *[8, 9] Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano topological space with an ideal $\mathcal{I}$ and A$\subseteq A^*_n$, then A^*_n=n-cl(A^*_n)=n-cl(A)*

Definition 2.3. *[8, 9] Let (U, $\mathcal{N}$) be an nano topological space eith an ideal $\mathcal{I}$ on U. The set operator n-cl* is called a nano $\star$ -closure and is defined as n-cl*(A)= A $\cup$ A^*_n for A $\subseteq$X.*

Theorem 2.4. *[8, 9] The set operator n-cl* satisfies the following conditions:*

1. A $\subseteq$ n-cl*(A).
2. n-cl*(ϕ) = ϕ and n-cl*(U) = U.
3. If A $\subseteq$B, then n-cl*(A) $\subseteq$ n-cl*(B).
4. n-cl*(A) $\cup$ n-cl*(B) = n-cl*(A $\cup$ B)
5. n-cl*(n-cl*(A)) =n-cl*(A)

Definition 2.5. *[8, 9] An ideal $\mathcal{I}$ ina space (U, $\mathcal{N}$, $\mathcal{I}$) is called $\mathcal{N}$-codense ideal if $\mathcal{N} \cap \mathcal{I} = \phi$.*

Definition 2.6. *[8, 9] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is n$\star$-dense in itself (resp. n$\star$-perfect and n$\star$-closed) if A $\subseteq A^*_n$ (resp. A= A^*_n, $A^*_n \subseteq$ A).*

Lemma 2.7. *[8, 9] Let (U, $\mathcal{N}$, $\mathcal{I}$) be a nano ideal topological space and A $\subseteq$ U. If A is n$\star$-dense in itself A^*_n =n-cl(A^*_n) =n-cl(A^*_n)=n-cl(A)=n-cl*(A).*

Definition 2.8. *[8, 9] A subset A of an nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be*

1. nano-$\mathcal{I}$-generalized closed (briefly, n$\mathcal{I}$g-closed if $A^*_n \subseteq$ V whenever A$\subseteq$ V and V is n-open.
2. n$\mathcal{I}$g-open if its complement is n$\mathcal{I}$g-closed.

Definition 2.9. *A subset M of a space (U, $\tau_R(X)$) is said to be nano semi-open set [7] if M $\subseteq$ Ncl(Nint(M)). The complement of nano semi-open set is called nano semi-closed set.*

Definition 2.10. *A subset M of a space (U, $\tau_R(X)$) is called*

1. nano semi- generalized closed set (briefly nsg-closed) [2] if Nscl(A) $\subseteq$ V whenever A $\subseteq$ V and V is nano semi open in (U, $\tau_R(X)$). The complement of nsg-closed set is called nsg-open set.

2. *nano g̈-closed set (briefly ng̈-closed) [3] if Ncl(A) $\subseteq$ V whenever A $\subseteq$ V and V is nsg-open in (U, $\tau_R(X)$). The complement of ng̈-closed set is called ng̈-open set.*

Definition 2.11. *[5] An nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is said to be a $nT_{\mathcal{I}}$-space if every $n\mathcal{I}_g$-closed subset of K is a n⋆-closed.*

Theorem 2.12. *[5] If (K, $\mathcal{N}$, $\mathcal{I}$) is a $nT_{\mathcal{I}}$ nano ideal space and A is an $n\mathcal{I}_g$-closed set, then A is n⋆-closed set.*

Lemma 2.13. *[5] If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then the following are equivalent*

1. $K = K_n^*$
2. $\mathcal{N} \cap \mathcal{I} = \phi$.
3. *If* $I \in \mathcal{I}$ *then n-int**(I) $= \phi$
4. *for every* $G \in \mathcal{N}$, $G \subseteq G_n^*$

Theorem 2.14. *[5] If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then the following are equivalent*

1. $K = K_n^*$
2. *for every* $A \in$ *Nano open,* $A \subseteq A_n^*$
3. *for every* $A \in$ *Nano semi open,* $A \subseteq A_n^*$

Remark 2.15.
1. *Every n-closed is n⋆-closed set but not conversely [1].*
2. *Every n-closed is nsg-closed set but not conversely [2].*
3. *Every n-closed set is ng̈-closed but not conversely [3].*
4. *Every ng̈-closed set is ng-closed but not conversely [3].*
5. *Every ng-closed set is $n\mathcal{I}_g$-closed but not conversely [9]*

Definition 2.16. *[1] A subset a of of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called an lightly nano $\mathcal{I}$-locally closed (briefly $\mathcal{L}$-n$\mathcal{I}$-LC) if A = M $\cap$ N where M is n-open and N is n⋆-closed.*

Definition 2.17. *A map f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) is said to be n⋆-continuous [6] (resp.$n\mathcal{I}_{\ddot{g}}$-continuous [4, 5], $\mathcal{L}$-n$\mathcal{I}$-LC-continuous[5]) if $f^{-1}(A)$ is n⋆-closed (resp. $n\mathcal{I}_{\ddot{g}}$-closed, $\mathcal{L}$-n$\mathcal{I}$-LC-set) in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$).*

3 n$\mathcal{I}_{\ddot{g}}$-closed sets

Definition 3.1. *A subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is said to be*

1. $n\mathcal{I}_{\ddot{g}}$*-closed if* $A_n^* \subseteq U$ *whenever* $A \subseteq U$ *and U is nsg-open,*
2. $n\mathcal{I}_{\ddot{g}}$*-open if its complement is* $n\mathcal{I}_{\ddot{g}}$*-closed.*

Theorem 3.2. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space, then every n$\mathcal{I}_{\ddot{g}}$-closed set is n$\mathcal{I}_g$-closed but not conversely.*

Proof. If A is a n$\mathcal{I}_{\ddot{g}}$-closed subset of (K, $\mathcal{N}$, $\mathcal{I}$) and U is any n-open set containing A, since every n-open set is nsg-open, we have U $\supseteq$ A^*_n. Hence A is n$\mathcal{I}$g-closed in (K, $\mathcal{N}$, $\mathcal{I}$).

Example 3.3. *Let K = {m, n, o, p}, with K/ R= {{o}, {p}, {m, o}} and X= {m, o}. Then the Nano topology $\mathcal{N}$ = {ϕ, {m}, {o}, {m, n}, K} and $\mathcal{I}$={$\emptyset$, {m}}. Then n$\mathcal{I}_{\ddot{g}}$-closed sets are ϕ, K, {m}, {n, p}, {m, n, p}, {n, o, p} and n$\mathcal{I}_g$-closed sets are ϕ, K, {m}, {n}, {p}, {m, n}, {m, p}, {n, o}, {n, p}, {o, p}, {m, n, o}, {m, n, p}, {m, o, p}, {n, o, p}. It is clear that {n} is n$\mathcal{I}_g$-closed but it is not n$\mathcal{I}_{\ddot{g}}$-closed.*

The following theorem gives characterizations of n$\mathcal{I}_{\ddot{g}}$-closed sets.

Theorem 3.4. *If (K, $\mathcal{N}$, $\mathcal{I}$) is any nano ideal topological space and A $\subseteq$ K, then the following are equivalent.*

1. *A is n$\mathcal{I}_{\ddot{g}}$-closed,*
2. *n-cl*(A)$\subseteq$U whenever A $\subseteq$ U and U is nsg-open in K,*
3. *For all k$\in$n-cl*(A), nsg-cl({k})$\cap$A$\neq\emptyset$.*
4. *n-cl*(A)$-$A contains no nonempty nsg-closed set,*
5. *$A^*_n$$-$A contains no nonempty nsg-closed set.*

Proof. (1)$\Rightarrow$(2) If A is n$\mathcal{I}_{\ddot{g}}$-closed, then $A^*_n$$\subseteq$U whenever A$\subseteq$U and U is nsg-open in K and so n-cl*(A)=A$\cup$$A^*_n$$\subseteq$U whenever A$\subseteq$U and U is nsg-open in K. This proves (2).

(2)$\Rightarrow$(3) Suppose k$\in$n-cl*(A). If nsg-cl({k})$\cap$A=$\emptyset$, then A$\subseteq$K$-$nsg-cl({k}). By (2), n-cl*(A)$\subseteq$ K$-$nsg-cl({k}), a contradiction, since k$\in$n-cl*(A).

(3)$\Rightarrow$(4) Suppose F$\subseteq$n-cl*(A)$-$A, F is nsg-closed and k$\in$F. Since F$\subseteq$K$-$A and F is nsg-closed, then A$\subseteq$K$-$F and F is nsg-closed, nsg-cl({k})$\cap$A=$\emptyset$. Since k$\in$n-cl*(A) by (3), nsg-cl({x})$\cap$A$\neq\emptyset$. Therefore n-cl*(A)$-$A contains no nonempty nsg-closed set.

(4)$\Rightarrow$(5) Since n-cl*(A)$-$A=(A$\cup$$A^*_n$)$-$A=(A$\cup$$A^*_n$)$\cap$$A^c$=(A$\cap$$A^c$)$\cup$ ($A^*_n$$\cap$$A^c$)=$A^*_n$$\cap$$A^c$= $A^*_n$$-$A. Therefore $A^*_n$$-$A contains no nonempty nsg-closed set.

(5)$\Rightarrow$(1) Let A$\subseteq$U where U is nsg-open set. Therefore K$-$U$\subseteq$K$-$A and so $A^*_n$$\cap$(K$-$U) $\subseteq$$A^*_n$$\cap$(K$-$A)=$A^*_n$$-$A. Therefore $A^*_n$$\cap$(K$-$U)$\subseteq$$A^*_n$$-$A. Since A^*_n is always n-closed set, so A^*_n is nsg-closed set and so $A^*_n$$\cap$(K$-$U) is a nsg-closed set contained in $A^*_n$$-$A. Therefore $A^*_n$$\cap$(K$-$U)=$\emptyset$ and hence $A^*_n$$\subseteq$U. Therefore A is n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 3.5. *Every n$\star$-closed set is n$\mathcal{I}_{\ddot{g}}$-closed but not conversely.*

Proof. Let A be a n$\star$-closed, then $A^*_n$$\subseteq$A. Let A$\subset$U where U is nsg-open. Hence $A^*_n$$\subseteq$U whenever A$\subseteq$U and U is nsg-open. Therefore A is n$\mathcal{I}_{\ddot{g}}$-closed.

Example 3.6. *Let K = {m, n, o, p} with K/ R= {{o}, {p}, {m, n}} and X= {n}. Then Nano topology $\mathcal{N}$ = {ϕ, {m, n}, K} and $\mathcal{I}$={$\emptyset$, {o}}. Then n$\mathcal{I}_{\ddot{g}}$-closed sets are ϕ, K, {o}, {o, p}, {m, o, p}, {n, o, p} and n$\star$-closed sets are ϕ, K, {o}, {o, p}. It is clear that {m, o, p} is n$\mathcal{I}_{\ddot{g}}$-closed set but it is not n$\star$-closed.*

Theorem 3.7. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. For every A∈$\mathcal{I}$, A is n$\mathcal{I}_{\ddot{g}}$-closed.*

Proof. Let A⊆U where U is nsg-open set. Since A_n^*=∅ for every A∈$\mathcal{I}$, then n-cl*(A)=A∪A_n^* =A⊆U. Therefore, by Theorem 3.4, A is n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 3.8. *If (K, $\mathcal{N}$, $\mathcal{I}$) is an nano ideal topological space, then A_n^* is always n$\mathcal{I}_{\ddot{g}}$-closed for every subset A of K.*

Proof. Let $A^*{}_n$⊆U where U is nsg-open. Since $(A_n^*)_n^*$⊆A_n^* Theorem 2.1 (4), we have $(A_n^*)_n^*$ ⊆U whenever A_n^*⊆U and U is nsg-open. Hence A_n^* is n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 3.9. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every n$\mathcal{I}_{\ddot{g}}$-closed, nsg-open set is n⋆-closed set.*

Proof. Since A is n$\mathcal{I}_{\ddot{g}}$-closed and nsg-open. Then A_n^*⊆A whenever A⊆A and A is nsg-open. Hence A is n⋆-closed.

Corollary 3.10. *If (K, $\mathcal{N}$, $\mathcal{I}$) is a n$T_{\mathcal{I}}$ nano ideal space and A is an n$\mathcal{I}_{\ddot{g}}$-closed set, then A is n⋆-closed set.*

Proof. By assumption A is n$\mathcal{I}_{\ddot{g}}$-closed in (K, $\mathcal{N}$, $\mathcal{I}$) and so by Theorem 3.2, A is n$\mathcal{I}_g$-closed. Since (K, $\mathcal{N}$, $\mathcal{I}$) is an n$T_{\mathcal{I}}$-space by Definition 2.11, A is n⋆-closed.

Corollary 3.11. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A be an n$\mathcal{I}_{\ddot{g}}$-closed set. Then the following are equivalent.*

1. *A is a n⋆-closed set,*
2. *n-cl*(A)−A is a nsg-closed set,*
3. *A_n^*−A is a nsg-closed set.*

Proof. (1)⇒(2) If A is n⋆-closed, then A_n^*⊆A and so n-cl*(A)−A=(A∪$A^*{}_n$)-*A* =∅. Hence n-cl*(A)−A is nsg-closed set.

(2)⇒(3) Since n-cl*(A)−A=A_n^*−A and so A_n^*−A is nsg-closed set.

(3)⇒(1) If A_n^*−A is a nsg-closed set, since A is n$\mathcal{I}_{\ddot{g}}$-closed set, by Theorem 3.4 (5), A_n^*−A=∅ and so A is n⋆-closed.

Theorem 3.12. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every n$\ddot{g}$-closed set is an n$\mathcal{I}_{\ddot{g}}$-closed set but not conversely.*

Proof. Let A be a n$\ddot{g}$-closed set. Then n-cl(A)⊆U whenever A⊆U and U is nsg-open. So by Theorem 2.1 (3), A_n^*⊆n-cl(A)⊆U whenever A⊆U and U is nsg-open. Hence A is n$\mathcal{I}_{\ddot{g}}$-closed.

Example 3.13. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 3.6. Then n$\ddot{g}$-closed sets are ∅, K, {o, p}, {m, o, p}, {n, o, p} . It is clear that {o} is n$\mathcal{I}_{\ddot{g}}$-closed set but it is not n$\ddot{g}$-closed.*

Theorem 3.14. *If (K, $\mathcal{N}$, $\mathcal{I}$) is an nano ideal topological space and A is a n⋆-dense in itself, n$\mathcal{I}_{\ddot{g}}$-closed subset of K, then A is n$\ddot{g}$-closed.*

Proof. Suppose A is a n⋆-dense in itself, n$\mathcal{I}_{\ddot{g}}$-closed subset of K. Let A⊆U where U is nsg-open. Then by Theorem 3.4 (2), n-cl*(A)⊆U whenever A⊆U and U is nsg-open. Since A is n⋆-dense in itself, by Lemma 2.7, n-cl(A)=n-cl*(A). Therefore n-cl(A)⊆U whenever A⊆U and U is nsg-open. Hence A is n$\ddot{g}$-closed.

Corollary 3.15. *If $(K, \mathcal{N}, \mathcal{I})$ is any nano ideal topological space where $\mathcal{I}=\{\emptyset\}$, then A is $n\mathcal{I}_{\breve{g}}$-closed if and only if A is $n\breve{g}$-closed.*

Proof. The proof follows from the fact that for $\mathcal{I}=\{\emptyset\}$, A^*_n=n-cl(A)$\supseteq$A. Therefore A is n$\star$-dense in itself. Since A is $n\mathcal{I}_{\breve{g}}$-closed, by Theorem 3.14, A is $n\breve{g}$-closed.

Conversely, by Theorem 3.12, every $n\breve{g}$-closed set is $n\mathcal{I}_{\breve{g}}$-closed set.

Theorem 3.16. *If $(K, \mathcal{N}, \mathcal{I})$ is any nano ideal topological space, then the following are equivalent*

1. *$K = K^*_n$ [5]*
2. *for every $A \in$ Nano open, $A \subseteq A^*_n$ [5]*
3. *for every $A \in$ nsg-open, $A \subseteq A^*_n$*

Proof. (2) $\Rightarrow$ (3). Suppose A $\in$ nsg-open $(K, \mathcal{N})$. Then there exists an n-open set M such that M $\subseteq$ A $\subseteq$ n-cl(M). Since M is n-open, M $\subseteq M^*_n$ and so by Lemma 2.2, A $\subseteq$ n-cl(M) $\subseteq$n-cl(M^*_n) = $M^*_n \subseteq A^*_n$. Hence A$\subseteq A^*_n$.
(3) $\Rightarrow$ (1). It is clear.

Corollary 3.17. *If $(K, \mathcal{N}, \mathcal{I})$ is any nano ideal topological space where $\mathcal{I}$ is $\mathcal{N}$-codense and A is a nsg-open, $n\mathcal{I}_{\breve{g}}$-closed subset of K, then A is $n\breve{g}$-closed.*

Proof. The proof follows Theorem 3.16, A is n$\star$-dense in itself. By Theorem 3.14, A is $n\breve{g}$-closed.

Theorem 3.18. *Every n-closed set is $n\mathcal{I}_{\breve{g}}$-closed but not conversely.*

Proof. Let A be a n-closed, then $A^*_n\subseteq$A. Let A$\subset$U where U is nsg-open. Hence $A^*_n\subseteq$U whenever A$\subseteq$U and U is nsg-open. Therefore A is $n\mathcal{I}_{\breve{g}}$-closed.

Example 3.19. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 3.3. Then n-closed sets are ϕ, K, {n, p}, {m, n, p}, {n, o, p}. It is clear that {m} is $n\mathcal{I}_{\breve{g}}$-closed set but it is not n-closed.*

Remark 3.20. *ng-closed sets and $n\mathcal{I}_{\breve{g}}$-closed sets are independent.*

Example 3.21. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 3.3. Then ng-closed sets are ϕ, K, {n}, {p}, {m, n}, {m, p}, {n, o}, {n, p}, {o, p}, {m, n, o}, {m, n, p}, {m, o, p}, {n, o, p}. It is clear that {o, p} is ng-closed set but it is not $n\mathcal{I}_{\breve{g}}$-closed. Also it is clear that {m} is $n\mathcal{I}_{\breve{g}}$-closed set but it is not ng-closed.*

Remark 3.22. *We have the following implications for the subsets stated above.*

$$\begin{array}{ccccc} n-closed & \longrightarrow & n\breve{g}-closed & \longrightarrow & ng-closed \\ \downarrow & & \downarrow & & \downarrow \\ n\star-closed & \longrightarrow & n\mathcal{I}_{\breve{g}}-closed & \longrightarrow & n\mathcal{I}_{g}-closed \end{array}$$

Theorem 3.23. *Let $(K, \mathcal{N}, \mathcal{I})$ be an nano ideal topological space and A$\subseteq$K. Then A is $n\mathcal{I}_{\breve{g}}$-closed if and only if A=F$-$M where F is n$\star$-closed and M contains no nonempty nsg-closed set.*

Proof. If A is n$\mathcal{I}_{\ddot{g}}$-closed, then by Theorem 3.4 (5), M=A_n^*−A contains no nonempty nsg-closed set. If F=ncl*(A), then F is n⋆-closed such that F−M=(A∪A_n^*)−(A_n^*−A)
=(A∪ A_n^*)∩ (A_n^*∩A^c)^c=(A∪A_n^*)∩ ((A_n^*)^c∪A)=(A∪A_n^*)∩(A∪(A_n^*)^c)=
A∪(A_n^*∩(A_n^*)^c)=A.

Conversely, suppose A=F−M where F is n⋆-closed and M contains no nonempty nsg-closed set. Let U be an nsg-open set such that A⊆U. Then F−M⊆U which implies that F∩(K−U)⊆M. Now A⊆F and F_n^*⊆F then A_n^*⊆F_n^* and so A_n^*∩(K−U)⊆F_n^*∩(K−U)⊆F∩ (K−U)⊆M. By hypothesis, since A_n^*∩(K−U) is nsg-closed, A_n^*∩(K−U)=∅ and so A_n^*⊆U. Hence A is n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 3.24. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. If A⊆B⊆A_n^*, then A_n^*=B_n^* and B is n⋆-dense in itself.*

Proof. Since A⊆B, then A_n^*⊆B_n^* and since B⊆A_n^*, then B_n^*⊆$(A_n^*)_n^*$⊆A_n^* Theorem 2.1(4). Therefore A_n^*=B_n^* and B⊆A_n^*⊆B_n^*. Hence proved.

Theorem 3.25. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A and B are subsets of K such that A⊆B⊆n-cl_n^*(A) and A is n$\mathcal{I}_{\ddot{g}}$-closed, then B is n$\mathcal{I}_{\ddot{g}}$-closed.*

Proof. Since A is n$\mathcal{I}_{\ddot{g}}$-closed, then by Theorem 3.4(1), n-cl_n^*(A)−A contains no nonempty nsg-closed set. Since n-cl*(B)−B⊆n-cl*(A)−A and so n-cl*(B)−B contains no nonempty nsg-closed set and so by Theorem 3.4 (4), B is n$\mathcal{I}_{\ddot{g}}$-closed.

Corollary 3.26. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A and B are subsets of K such that A⊆ B⊆A_n^* and A is n$\mathcal{I}_{\ddot{g}}$-closed, then A and B are n$\ddot{g}$-closed sets.*

Proof. Let A and B be subsets of K such that A⊆B⊆A_n^* which implies that A⊆B ⊆A_n^*⊆n-cl*(A) and A is n$\mathcal{I}_{\ddot{g}}$-closed. By Theorem 3.25, B is n$\mathcal{I}_{\ddot{g}}$-closed. Since A⊆B⊆A_n^*, then A_n^*=B_n^* and so A and B are n⋆-dense in itself. By Theorem 3.14, A and B are n$\ddot{g}$-closed.

The following theorem gives a characterization of n$\mathcal{I}_{\ddot{g}}$-open sets.

Theorem 3.27. *Let (K, $\mathcal{N}$, $\mathcal{I}$)be an nano ideal topological space and A⊆K. Then A is n$\mathcal{I}_{\ddot{g}}$-open if and only if F⊆n-int*(A) whenever F is nsg-closed and F⊆A.*

Proof. Suppose A is n$\mathcal{I}_{\ddot{g}}$-open. If F is nsg-closed and F⊆A, then K−A⊆K−F and so n-cl*(K−A)⊆K−F by Theorem 3.4 (2). Therefore F⊆K−n-cl*(K−A)=n-int*(A). Hence F⊆n-int*(A).

Conversely, suppose the condition holds. Let U be a nsg-open set such that K−A⊆U. Then K−U⊆A and so K−U⊆n-int*(A). Therefore n-cl*(K−A)⊆U. By Theorem 3.4 (2), K−A is n$\mathcal{I}_{\ddot{g}}$-closed. Hence A is n$\mathcal{I}_{\ddot{g}}$-open.

Corollary 3.28. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. If A is n$\mathcal{I}_{\ddot{g}}$-open, then F⊆n-int*(A) whenever F is n-closed and F⊆A.*

The following theorem gives a property of n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 3.29. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. If A is n$\mathcal{I}_{\ddot{g}}$-open and n-int*(A)⊆B⊆A, then B is n$\mathcal{I}_{\ddot{g}}$-open.*

Proof. Since A is n$\mathcal{I}_{\ddot{g}}$-open, then K−A is n$\mathcal{I}_{\ddot{g}}$-closed. By Theorem 3.4 (4), n-cl*(K−A)− (K−A) contains no nonempty nsg-closed set. Since n-int*(A)⊆n-int*(B) which implies that n-cl*(K−B)⊆n-cl*(K−A) and so n-cl*(K−B)−(K−B)⊆ n-cl*(K−A)−(K−A). Hence B is n$\mathcal{I}_{\ddot{g}}$-open.

The following theorem gives a characterization of n$\mathcal{I}_{\ddot{g}}$-closed sets in terms of n$\mathcal{I}_{\ddot{g}}$-open sets.

Theorem 3.30. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A⊆K. Then the following are equivalent.*

1. *A is n$\mathcal{I}_{\ddot{g}}$-closed,*
2. *A∪(K−A^*_n) is n$\mathcal{I}_{\ddot{g}}$-closed,*
3. *A^*_n−A is n$\mathcal{I}_{\ddot{g}}$-open.*

Proof. (1)⇒(2) Suppose A is n$\mathcal{I}_{\ddot{g}}$-closed. If U is any nsg-open set such that A∪(K−A^*_n)⊆U, then K−U⊆ K−(A∪(K−A^*_n))=K∩(A∪(A^*_n)^c)^c = A^*_n∩A^c=A^*_n−A.
Since A is n$\mathcal{I}_{\ddot{g}}$-closed, by Theorem 3.4 (5), it follows that K−U=∅ and so K=U. Therefore A∪(K−A^*_n)⊆U which implies that A∪(K−A^*_n)⊆K
and so (A∪(K−A^*_n))$^*_n$⊆K^*_n⊆K=U. Hence A∪(K−A^*_n) is n$\mathcal{I}_{\ddot{g}}$-closed.

(2)⇒(1) Suppose A∪(K−A^*_n) is n$\mathcal{I}_{\ddot{g}}$-closed. If F is any nsg-closed set such that F⊆A^*_n−A, then F⊆A^*_n and F⊈A which implies that K−A^*_n⊆K−F and A⊆K−F.
Therefore A∪(K−A^*_n)⊆A∪(K−F)=K−F and K−F is nsg-open. Since (A∪(K−A^*_n))$^*_n$ ⊆K−F which implies that A^*_n∪(K−A^*_n)$^*_n$⊆K−F and so A^*_n⊆K−F which implies that F⊆K−A^*_n. Since F⊆A^*_n, it follows that F=∅. Hence A is n$\mathcal{I}_{\ddot{g}}$-closed.

(2)⇔(3) Since K−(A^*_n−A)=K∩(A^*_n∩A^c)^c=K∩((A^*_n)^c∪A)=(K∩ (A^*_n)^c)∪(K∩A)=A∪ (K−A^*_n) is n$\mathcal{I}_{\ddot{g}}$-closed. Hence A^*_n−A is n$\mathcal{I}_{\ddot{g}}$-open.

Theorem 3.31. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. Then every subset of K is n$\mathcal{I}_{\ddot{g}}$-closed if and only if every nsg-open set is n⋆-closed.*

Proof. Suppose every subset of K is n$\mathcal{I}_{\ddot{g}}$-closed. If U⊆K is nsg-open, then U is n$\mathcal{I}_{\ddot{g}}$-closed and so U^*_n⊆U. Hence U is n⋆-closed.

Conversely, suppose that every nsg-open set is n⋆-closed. If U is nsg-open set such that A⊆U⊆K, then A^*_n⊆U^*_n⊆U and so A is n$\mathcal{I}_{\ddot{g}}$-closed.

4 sg-n$\mathcal{I}$-locally closed sets

We introduce the following definition.

Definition 4.1. *A subset a of of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called an sg-n$\mathcal{I}$-locally closed set(briefly sg-n$\mathcal{I}$-LC) if A = M ∩ N where M is nsg-open and N is n⋆-closed.*

Proposition 4.2. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space and A a subset of K. Then the following hold.*

1. *If A is nsg-open , then A is sg-n$\mathcal{I}$-LC set.*

2. *A is n⋆-closed, then A is sg-n$\mathcal{I}$-LC set.*

3. *If A is a $\mathcal{L}$-n$\mathcal{I}$-LC-set, then A is an sg-n$\mathcal{I}$-LC set.*

Proof. It is obvious from Definition 2.16 and 4.1.
The converse of the above Proposition 4.2 need not be true as shown in the following examples.

Example 4.3. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 3.3. Then nsg-open sets are ϕ, K, {m}, {o}, {m, n}, {m, o}, {m, p}, {n, o}, {o, p}, {m, n, o}, {m, n, p}, {m, o, p}, {n, o, p}, sg-n$\mathcal{I}$-LC sets are power set of K, n⋆-closed sets are ϕ, K, {m}, {n, p}, {m, n, p}, {n, o, p} and $\mathcal{L}$-n$\mathcal{I}$-LC-set are ϕ, K, {m}, {o}, {m, o}, {n, p}, {m, n, p}, {n, o, p}. It is clear that {m, o} is sg-n$\mathcal{I}$-LC set but it is not n⋆-closed. It is clear that {n} is an sg-n$\mathcal{I}$-LC set but it is not nsg-open. Also It is clear that {m, n} is sg-n$\mathcal{I}$-LC set but it is not $\mathcal{L}$-n$\mathcal{I}$-LC-set.*

Theorem 4.4. *Let (K, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space. If A is an sg-n$\mathcal{I}$-LC-set and B is a n⋆-closed set, then A ∩ B is an sg-n$\mathcal{I}$-LC-set.*

Proof. Let B be n⋆-closed , then A ∩ B = (O ∩ P) ∩ B = O ∩ (P ∩ B), where P ∩ B is n⋆-closed. Hence A ∩ B is an sg-n$\mathcal{I}$-LC-set.

Theorem 4.5. *A subset of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is n⋆-closed if and only if it is*

1. *$\mathcal{L}$-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{\ddot{g}}$-closed.*

2. *sg-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{\ddot{g}}$-closed.*

Proof. (1) Necessity is trivial. We prove only sufficiency. Let A be $\mathcal{L}$-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{\ddot{g}}$-closed set. Since A is $\mathcal{L}$-n$\mathcal{I}$-LC set, A = O ∩ P , where O is n-open and P is n⋆-closed. So we have A = O ∩ P ⊆ O. Since A is n$\mathcal{I}_{\ddot{g}}$-closed, $A^*_n \subseteq$ O. Also since A = O ∩ P ⊆ P and P is n⋆-closed, we have $A^*_n \subseteq$ P. Consequently, $A^*_n \subseteq$ O ∩ P = A and hence A is n⋆-closed.

(2) Necessity is trivial. We prove only sufficiency. Let A be sg-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{\ddot{g}}$-closed set. Since A is sg-n$\mathcal{I}$-LC set, A = O ∩ P , where O is nsg -open and P is n⋆-closed. So we have A = O ∩ P ⊆ O. Since A is n$\mathcal{I}_{\ddot{g}}$-closed, $A^*_n \subseteq$ O. Also since A = O ∩ P ⊆ P and P is n⋆-closed, we have $A^*_n \subseteq$ P. Consequently, $A^*_n \subseteq$ O ∩ P = A and hence A is n⋆-closed.

Remark 4.6.

1. *The notions of $\mathcal{L}$-n$\mathcal{I}$-LC set and n$\mathcal{I}_{\ddot{g}}$-closed set are independent.*

2. *The notions of sg-n$\mathcal{I}$-LC-set and n$\mathcal{I}_{\ddot{g}}$-closed set are independent.*

Example 4.7. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be defined as an Example 3.6. Then sg-n$\mathcal{I}$-LC-sets are ϕ, K, {m}, {n}, {o}, {p}, {m, n}, {o, p}, {m, n, o}, {m, n, p} and $\mathcal{L}$-n$\mathcal{I}$-LC set are ϕ, K, {o}, {m, n}, {o, p}. (1) It is clear that {m} is sg-n$\mathcal{I}$-LC-set but it is not n$\mathcal{I}_{\ddot{g}}$-closed. Also it is clear that {m, o, p} is an n$\mathcal{I}_{\ddot{g}}$-closed but it is not sg-n$\mathcal{I}$-LC-set. (2) It is clear that {m, n} is $\mathcal{L}$-n$\mathcal{I}$-LC-set but it is not n$\mathcal{I}_{\ddot{g}}$-closed. Also it is clear that {n, o, p} is an n$\mathcal{I}_{\ddot{g}}$-closed but it is not $\mathcal{L}$-n$\mathcal{I}$-LC-set.*

Definition 4.8. *[3] Let A be a subset of a nano topological space (K, $\mathcal{N}$). Then the Nano sg-kernel of the set A, denoted by nsg-ker(A), is the intersection of all nsg-open supersets of A.*

Definition 4.9. *A subset A of an nano ideal topological space (K, $\mathcal{N}$, $\mathcal{I}$) is called n$\wedge_{sg}$-set if A = nsg-ker(A).*

Definition 4.10. *A subset A of an nano ideal topological space* $(K, \mathcal{N}, \mathcal{I})$ *is called* η_{sg}*-n*$\mathcal{I}$*-closed if* $A = R \cap S$ *where R is a n*$\wedge_{sg}$*-set and S is a n*$\star$*-closed.*

Lemma 4.11. 1. *Every n*$\star$*-closed set is* η_{sg}*-n*$\mathcal{I}$*-closed but not conversely.*

2. *Every n*$\wedge_{sg}$*-set is* η_{sg}*-n*$\mathcal{I}$*-closed but not conversely.*

Proof. 1. Follows from Definitions 2.6 and 4.10.

2. Follows from Definitions 2.6 and 4.10.

Remark 4.12. *The concepts of n*$\star$*-closed and n*$\wedge_{sg}$*-set are independent.*

Example 4.13. *Let K ,* $\mathcal{N}$ *and* $\mathcal{I}$ *be as in the Example 4.3,* η_{sg}*-n*$\mathcal{I}$*-closed sets are power set of K and n*$\wedge_{sg}$*-sets are* ϕ*, K, {m}, {o}, {m, n}, {m, o}, {m, p}, {n, o}, {o, p}, {m, n, o}, {m, n, p}, {m, o, p}, {n, o, p}. (1) It is clear that {n} is* η_{sg}*-n*$\mathcal{I}$*-closed but it is not n*$\star$*-closed. (2) It is clear that {p} is* η_{sg}*-n*$\mathcal{I}$*-closed but it is not n*$\wedge_{sg}$*-set. (3) It is clear that {o} is n*$\wedge_{sg}$*-set but it is not n*$\star$*-closed. Also it is clear that {n, p} is n*$\star$*-closed set but it is not n*$\wedge_{sg}$*-set.*

Lemma 4.14. *For a subset A of an nano ideal topological space* $(K, \mathcal{N}, \mathcal{I})$ *the following are equivalent.*

1. *A is* η_{sg}*-n*$\mathcal{I}$*-closed.*
2. $A = O \cap$ *n-cl***(A) where O is a n*$\wedge_{sg}$*-set .*
3. *A= nsg-ker(A)* $\cap$ *n-cl***(A)*

(1) $\Rightarrow$ (2). Let A be a η_{sg}-n$\mathcal{I}$-closed set. Then A = O $\cap$ P where O is a η_{sg}-n$\mathcal{I}$-closed set and P is a n$\star$-closed. Clearly A $\subseteq$ O $\cap$ n-cl*(A). Since P is a n$\star$-closed, n-cl*(A) $\subseteq$ n-cl*(P) = P and so O $\cap$ n-cl*(A) $\subseteq$ O $\cap$ P = A. Therefore, A = O $\cap$ n-cl*(A).
(2) $\Rightarrow$ (3). Let A = O $\cap$ n-cl*(A), where O is a n$\wedge_{sg}$-set. Since O is a n$\wedge_{sg}$-set, we have A= nsg-ker(A) $\cap$ n-cl*(A).
(3) $\Rightarrow$ (1). Let A= nsg-ker(A) $\cap$ n-cl*(A). By Definitions 4.9 and 4.10 and the notion of n$\star$-closed set, we get A is η_{sg}-n$\mathcal{I}$-closed.

Lemma 4.15. *A subset A* $\subseteq$ $(K, \mathcal{N}, \mathcal{I})$ *is n*$\mathcal{I}_{\ddot{g}}$*-closed if and only if n-cl***(A)* $\subseteq$ *nsg-ker(A).*

Proof. Suppose that A $\subseteq$ K is an n$\mathcal{I}_{\ddot{g}}$-closed set. Suppose k $\notin$ nsg-ker(A). Then there exists an nsg-open set U containing A such that k $\notin$ U. Since A is an n$\mathcal{I}_{\ddot{g}}$-closed set, A $\subseteq$ U and U is nsg-open implies that n-cl*(A) $\subseteq$ U and so k $\notin$ n-cl*(A). Therefore n-cl*(A) $\subseteq$ nsg-ker(A).
Conversely, suppose n-cl*(A) $\subseteq$ nsg-ker(A). If A $\subseteq$ U and U is nsg-open, then n-cl*(A) $\subseteq$ nsg-ker(A) $\subseteq$ U . Therefore, A is n$\mathcal{I}_{\ddot{g}}$-closed.

Theorem 4.16. *For a subset A of an nano ideal topological space* $(K, \mathcal{N}, \mathcal{I})$ *the following are equivalent.*

1. *A is n*$\star$*-closed.*
2. *A is n*$\mathcal{I}_{\ddot{g}}$*-closed and sg-n*$\mathcal{I}$*-LC.*
3. *A is n*$\mathcal{I}_{\ddot{g}}$*-closed and* η_{sg}*-n*$\mathcal{I}$*-closed.*

Proof. (1) ⇒ (2) ⇒ (3) Obvious.
(3) ⇒ (1). Since a is n$\mathcal{I}_{\ddot{g}}$-closed, by (2), Lemma 4.15, n-cl*(A) ⊆ nsg-ker(A). Since A is η_{sg}-n$\mathcal{I}$-closed, by Lemma 4.14, A = nsg-ker(A) ∩ n-cl*(A)= n-cl*(A). Hence A is n⋆-closed.

Remark 4.17. *The concepts of n$\mathcal{I}_{\ddot{g}}$-closedness and η_{sg}-n$\mathcal{I}$-closedness are independent.*

Example 4.18. *Let K , $\mathcal{N}$ and $\mathcal{I}$ be as in the Example 3.6, η_{sg}-n$\mathcal{I}$-closed sets are ϕ, K, {m}, {n}, {m, n}, {m, n, o}, {m, n, p}. It is clear that {m, n} is η_{sg}-n$\mathcal{I}$-closed but it is not n$\mathcal{I}_{\ddot{g}}$-closed. Also it is clear that {o} is n$\mathcal{I}_{\ddot{g}}$-closed set but it is not η_{sg}-n$\mathcal{I}$-closed.*

5 Decompositions of nano ⋆-continuity

Definition 5.1. *A map f: (K, $\mathcal{N}$, $\mathcal{I}$) → (L, $\mathcal{N}'$) is said to be sg-n$\mathcal{I}$-LC-continuous (resp. η_{sg}-n$\mathcal{I}$-continuous) if $f^{-1}(A)$ is sg-n$\mathcal{I}$-LC-set(resp. η_{sg}-n$\mathcal{I}$-closed) in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$).*

Theorem 5.2. *A map f: (K, $\mathcal{N}$, $\mathcal{I}$) → (L, $\mathcal{N}'$) is n⋆-continuous if and only if it is*

1. *$\mathcal{L}$-n$\mathcal{I}$-LC-continuous and n$\mathcal{I}_{\ddot{g}}$-continuous.*
2. *sg-n$\mathcal{I}$-LC-continuous and n$\mathcal{I}_{\ddot{g}}$-continuous.*

Proof. It is an immediate consequence of Theorem 4.5.

Theorem 5.3. *A map f: (K, $\mathcal{N}$, $\mathcal{I}$) → (L, $\mathcal{N}'$) the following are equivalent.*

1. *f is n⋆-continuous.*
2. *f is n$\mathcal{I}_{\ddot{g}}$–continuous and sg-n$\mathcal{I}$-LC-continuous.*
3. *f is n$\mathcal{I}_{\ddot{g}}$-continuous and η_{sg}-n$\mathcal{I}$-continuous.*

Proof. It is an immediate consequence of Theorem 4.16.

Conclusion

General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc. In this paper, we have defined and studied the notions of n$\mathcal{I}_{\ddot{g}}$-closed and n$\mathcal{I}_{\ddot{g}}$–continuous map in nano ideal topological and discussed their properties. Also we have discussed the relationships between the other existing continuities. Finally, we have found a decomposition of nano ⋆-continuity using n$\mathcal{I}_{\ddot{g}}$-continuous map. In future, we have extend this work in various nano ideal topological fields.

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