

## Anisotropy of Some Hexagonal Systems

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### Abstract:

The norm of elastic constant tensor and the norms of the irreducible parts of the elastic constants of some hexagonal systems (Apatite Fluor apatite, Hydroxyapatite, Beryl, Beryllium, Beryllium oxide, Biotite, Cadmium, Cadmium selenide, Cadmium sulfide, Cadmium telluride, Cesium copper chloride, Calcium-magnesium, Cancrinite, Cerium fluoride, Cobalt, Cobalt nickel, Co – 32 wt % Ni, Copper chloride, Dunite, Dysprosium, Erbium, Gadolinium, Gadolinium-yttrium, Gd – 40 at % Y, Gallium selenide, Graphite, Hafnium, Holmium, Indium bismuth, Indium selenide, Lead germinate vanadate, Lithium iodate, Lithium iodate, Yttrium, Zinc oxide (Zincite), Zinc selenide, and Zinc telluride) are calculated. The relation of the scalar parts norm and the other parts norms and the anisotropy of the materials are presented. The norm ratios are used as a criterion to present the anisotropy degree of the properties of these materials.

**Key Words:** Norm, Anisotropy, Elastic Constant, Irreducible, and hexagonal systems.

### 1 – Introduction:

The hexagonal system has four crystallographic axes consisting of three equal horizontal, or equilateral axes at 120 degrees to each other, as well as one vertical axis which is perpendicular to the other three. The decomposition procedure and the decomposition of elastic constant tensor (Elastic constant tensor can be decomposed into two scalar parts, two deviator parts and one nonor part) is given in [1,2,3,4], also the definition of norm concept and the norm ratios and the relationship between the anisotropy and the norm ratios are given in [1,2,3,4]. As the ratio  $N_s / N$  (Norm of the scalar part of the elastic constant tensor/Norm of the elastic constant tensor) becomes close to one the material becomes more isotropic, and as the sum of the ratios  $N_d / N$  (Norm of the deviator part of the elastic constant tensor/Norm of the elastic constant tensor) and  $N_n / N$  (Norm of the nonor part of the elastic constant tensor/Norm of the elastic constant tensor) becomes close to one the material becomes more anisotropic as explained in [1,2,3,4].

### 2 – Calculations:

The elastic constants of Hexagonal Systems Materials are given in the following table, [5].

**Table 1. Elastic constants in GPa**

Material	$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$
Apatite Fluor apatite $Ca_{10}(PO_4)_6F_2$	139	178	44.3	45	56
Hydroxyapatite $Ca_{10}(PO_4)_6(OH)_2$	139.9	179.5	36.2	12.9	69.0
Beryl, $Be_2Al_2Si_6O_{18}$	290	257	67.7	107	83
Material	$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$

Beryllium, Be	292	349	163	24	6
Beryllium oxide, (piezoel), BeO	470	494	153	168	119
Biotite $K(Mg,Fe)_3AlSi_3O_{10}(OH,F)_2$	186	54	5.8	32	12
Cadmium, Cd	116	50.9	19.6	42	41
Cadmium selenide, (piezoel), CdSe	74.1	84.3	13.4	45.2	38.9
Cadmium sulfide, (piezoel), CdS	87.0	94.1	14.9	54.6	47.5
Cadmium telluride, (piezoel), CdTe	62.2	68.9	11.6	35.9	29.1
Cesium copper chloride (piezoel), CsCuCl <sub>3</sub>	29.0	46.6	5.49	11.3	10.3
Calcium-magnesium CaMg <sub>3</sub>	56.2	61.6	18.0	15.9	15
Cancrinite (piezoel), $(Na_2Ca)_4(AlSiO_4)_6CO_3.nH_2O$	52	83	24	2	12
Cerium fluoride, CeF <sub>3</sub>	180	225	34.2	88	64
Cobalt, Co	295	335	71.0	159	111
Cobalt nickel Co – 32 wt % Ni	326	358	74.0	161	95
Copper chloride, CuCl	52.5	61.6	7.0	41.3	32.2
Dunite	198	238	67	76	96
Dysprosium, Dy	74.0	78.6	24.3	25.5	21.8
Erbium, Er	84.1	84.7	27.4	29.4	22.6
Gadolinium, Gd	66.7	71.9	20.1	25.0	21.3
Gadolinium-yttrium Gd – 40 at % Y	67.9	72.6	22.5	25.1	16.2
Gallium selenide (piezoel), GaSe	111	35.3	10.2	33	12
Graphite, C	1060	36.5	4	180	15
Hafnium, Hf	181	197	55.7	77	66
Holmium, Ho	76.5	79.6	25.9	25.6	21.0
Indium bismuth, In <sub>2</sub> Bi	49.2	54.1	9.66	39.6	29.0
Indium selenide, InSe	118	38.2	11.7	47.5	32
Lead germinate vanadate $Pb_5(GeO_4)(VO_4)_2$	71	84	17	21	33
Lead silicate vanadate $Pb_5(SiO_4)(VO_4)_2$	77	92	21	25	36

Material	$C_{11}$	$C_{33}$	$C_{44}$	$C_{12}$	$C_{13}$
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Lithium iodate (piezoel) LiIO <sub>3</sub>	81.2	52.9	17.8	31.8	9.2
Yttrium, Y	77.9	76.9	24.3	29.2	20
Zinc oxide (Zincite) (piezoel), ZnO	209	218	44.1	120	104
Zinc selenide, ZnSe	107	116	25.0	45	35
Zinc telluride, ZnTe	86	93	20.2	37	30

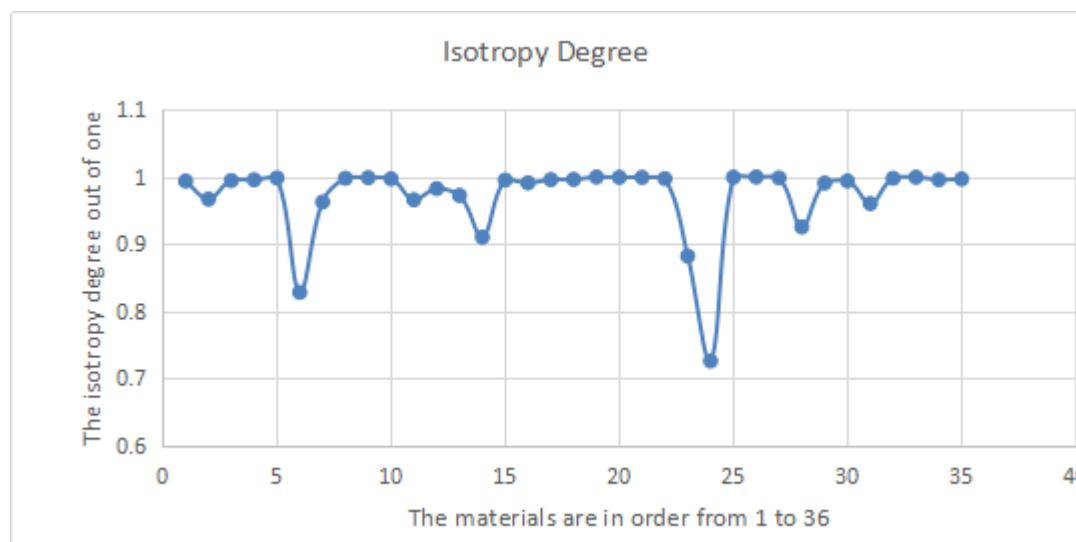
By using table 1, the decomposition of the elastic constant tensor and the norm concept we can calculate the norms and the norm ratios of the given materials as in the following table.

**Table 2. The norms and norm ratios (the anisotropy degree)**

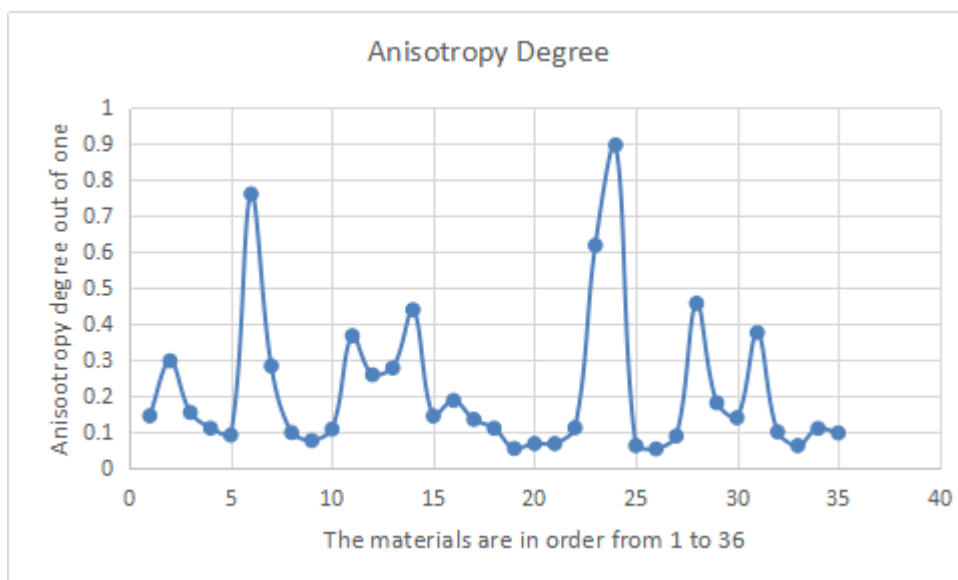
Material	$N_s$	$N_d$	$N_n$	$N$	$N_s / N$	$N_d / N$	$N_n / N$	Sum of 7 <sup>th</sup> and 8 <sup>th</sup> columns
Apatite Fluor apatite Ca <sub>10</sub> (PO <sub>4</sub> ) <sub>6</sub> F <sub>2</sub>	300.3 863	33.43 556	10.686 02	302.43 03	0.9932 42	0.1105 56	0.0353 34	0.14589
Hydroxyapatite Ca <sub>10</sub> (PO <sub>4</sub> ) <sub>6</sub> (OH) <sub>2</sub>	299.0 55	78.27 699	14.068 65	309.44 97	0.9664 09	0.2529 55	0.0454 63	0.298418
Beryl, Be <sub>2</sub> Al <sub>2</sub> Si <sub>6</sub> O <sub>18</sub>	536.9 017	41.44 993	42.359 7	540.16 29	0.9939 63	0.0767 36	0.0784 2	0.155156
Beryllium , Be	604.7 319	58.88 834	8.8409 53	607.65 67	0.9951 87	0.0969 11	0.0145 49	0.11146
Beryllium oxide, (piezoel), BeO	917.5 332	41.10 415	43.820 38	919.49 82	0.9978 63	0.0447 03	0.0476 57	0.09236
Biotite K(Mg,Fe) <sub>3</sub> AlSi <sub>3</sub> O <sub>10</sub> (O H,F) <sub>2</sub>	227.8 67	135.3 358	74.110 25	275.19 35	0.8280 25	0.4917 84	0.2693 02	0.761086
Cadmium, Cd	196.6 168	55.56 432	2.4985 3	204.33 26	0.9622 39	0.2719 31	0.0122 28	0.284159
Cadmium selenide, (piezoel), CdSe	167.6 308	6.327 031	10.378 51	168.07 09	0.9973 81	0.0376 45	0.0617 51	0.099396
Cadmium sulfide, (piezoel), CdS	197.4 217	5.107 657	10.186 32	197.75 03	0.9983 38	0.0258 29	0.0515 11	0.07734
Cadmium telluride,	135.2 464	4.594 44	10.186 32	135.70 72	0.9966 04	0.0338 56	0.0750 61	0.108917

(piezoel), CdTe								
Cesium copper chloride (piezoel), CsCuCl <sub>3</sub>	63.28 701	11.41 405	12.700 22	65.550 14	0.9654 75	0.1741 27	0.1937 48	0.367875
Calcium-magnesium CaMg <sub>3</sub>	100.5 101	10.11 704	16.451 86	102.34 89	0.9820 34	0.0988 49	0.1607 43	0.259592
Cancrinite (piezoel), (Na <sub>2</sub> Ca) <sub>4</sub> (AlSiO <sub>4</sub> ) <sub>6</sub> CO 3.nH <sub>2</sub> O	115.7 857	27.43 403	5.7658 39	119.13 1	0.9719 19	0.2302 84	0.0483 99	0.278683
Cerium fluoride, CeF <sub>3</sub>	289.0 439	131.5 104	8.2643 69	317.66 28	0.9099 08	0.4139 94	0.0260 16	0.44001
Cobalt, Co	619.3 554	42.89 665	47.664 27	622.66 62	0.9946 83	0.0688 92	0.0765 49	0.145441
Cobalt nickel Co – 32 wt % Ni	644.0 702	46.54 75	76.109 07	650.21 97	0.9905 42	0.0715 87	0.1170 51	0.188638
Copper chloride, CuCl	128.7 497	9.246 621	8.3412 47	129.35 05	0.9953 55	0.0714 85	0.0644 86	0.135971
Dunite	443.0 747	40.34 747	9.2253 42	445.00 36	0.9956 65	0.0906 68	0.0207 31	0.111399
Dysprosium, Dy	147.3 096	3.616 671	4.5357 93	147.42 38	0.9992 25	0.0245 32	0.0307 67	0.055299
Erbium, Er	163.6 616	5.901 553	5.3814 5	163.85 63	0.9988 11	0.0360 17	0.0328 42	0.068859
Gadolinium, Gd	133.8 473	3.198 517	5.9964 72	134.01 97	0.9987 13	0.0238 66	0.0447 43	0.068609
Gadolinium-yttrium Gd – 40 at % Y	132.7 145	8.074 449	6.9574 46	133.14 19	0.9967 91	0.0606 45	0.0522 56	0.112901
Gallium selenide (piezoel), GaSe	149.4 059	73.47 677	31.327 72	169.41 78	0.8818 78	0.4337 02	0.1849 14	0.618616
Graphite, C	1125. 582	986.3 977	403.80 09	<b>1550.</b> <b>151</b>	<b>0.726</b> <b>111</b>	0.6363 23	0.2604 91	<b>0.896814</b>
Hafnium, Hf	374.9 283	14.79 342	8.9178 31	375.32 6	0.9989 4	0.0394 15	0.0237 6	0.063175
Holmium, Ho	150.7 527	4.129 77	4.0360 87	150.86 33	<b>0.999</b> <b>267</b>	0.0273 74	0.0267 53	<b>0.054127</b>
Indium bismuth, In <sub>2</sub> Bi	131.3 914	6.713 308	5.1277 53	131.66 27	0.9979 4	0.0509 89	0.0389 46	0.089935
Indium selenide, InSe	181.8 634	72.56 688	17.451 27	196.58 28	0.9251 24	0.3691 42	0.0887 73	0.457915

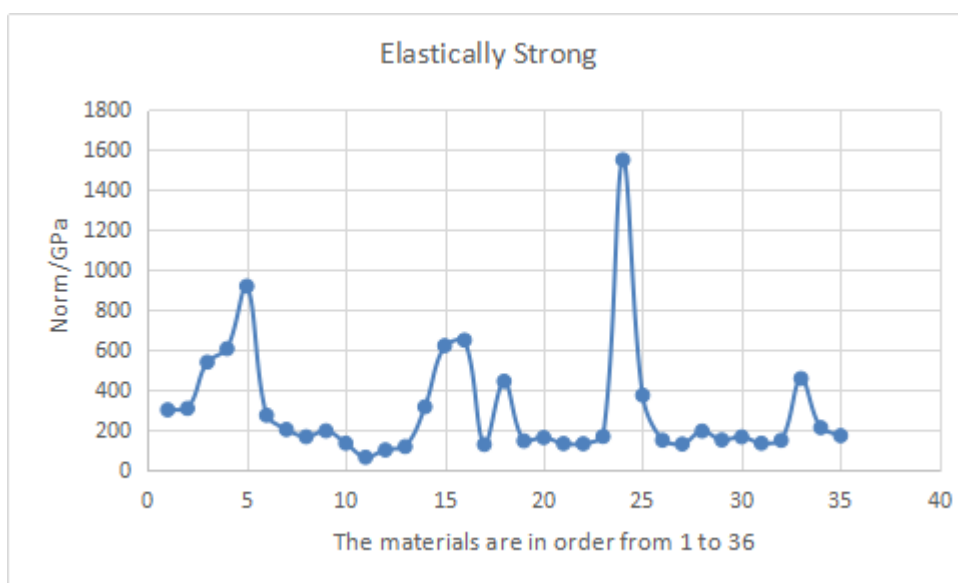
Lead germinate vanadate $Pb_5(GeO_4)(VO_4)_2$	150.5 311	19.57 89	8.0721 74	152.01 36	0.9902 48	0.1287 97	0.0531 02	0.181899
Lead silicate vanadate $Pb_5(SiO_4)(VO_4)_2$	166.1 418	18.49 729	4.9970 6	167.24 3	0.9934 16	0.1106 01	0.0298 79	0.14048
Lithium iodate (piezoel) $LiIO_3$	130.1 032	33.95 166	17.105 32	135.54 39	0.9598 6	0.2504 85	0.1261 98	0.376683
Yttrium, Y	149.8 293	8.478 225	6.7652 51	150.22 14	0.9973 9	0.0564 38	0.0450 35	0.101473
Zinc oxide (Zincite) (piezoel), ZnO	458.0 924	12.58 058	16.374 98	458.55 76	0.9989 86	0.0274 35	0.0357 1	0.063145
Zinc selenide, ZnSe	212.3 666	3.297 082	20.372 63	213.36 7	0.9953 11	0.0154 53	0.0954 82	0.110935
Zinc telluride, ZnTe	172.7 437	2.377 703	14.683 67	173.38 3	0.9963 13	0.0137 14	0.0846 89	0.098403



Graph 1. The Isotropy Degree of the Given Materials.



Graph 2. The Anisotropy Degree of the Given Materials.



Graph 3. The Elastically Strong of the Given Materials.

### 3 - Results and Conclusion:

From table 2 and the Graphs (Graph 1 to Graph 3), and analyzing the ratio  $N_s / N$  we can conclude that Holmium, Ho is the most isotropic material with highest value of  $N_s / N$  (0.999267) and lowest sum value of  $N_d / N$  and  $N_n / N$  (0.054227), and Graphite, C is the most anisotropic material with highest sum value of  $N_d / N$  and  $N_n / N$  (0.896814) and with lowest value of  $N_s / N$  (0.726111), because for isotropic material  $N_s / N = 1$ , and  $N_d / N = 0$  and  $N_n / N = 0$ . Which means that as values of  $N_d / N$  and  $N_n / N$  increase the anisotropy increases. And also the elastically strongest material is Graphite, C, which has the highest value of  $N$  (1550.151).

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