The Basic Law of Dynamics and The Mach Principle

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Abstract

The dependences of inertial and gravimetric masses from each other are derived taking into account the response of the Universe to interacting bodies. It was found that the inertial mass depends not only on the size of the corresponding gravitational mass, but also on the force acting on the body, and the relations of the inert and gravitational masses depend not only on the force acting on the body, but also on the gravitational mass of this body. However, if instead of the ratio of inert and gravitational masses we choose the difference of the inverse values of these masses, the resulting formula turns out to be independent of the gravitational mass of the body. A formula for the basic law of dynamics is proposed taking into account the response of the Universe to interacting bodies in accordance with the Mach principle. It is shown that the error from the use of Newton's second law without taking into account the response of the Universe to interacting bodies in many experiments can exceed the error caused by other factors.

Keywords: Basic Law of Dynamics, Newton's Second Law, Mach Principle, Equivalence Principle, Inert Mass, Gravitational Mass.

1. Introduction

In the case of the contact interaction of two solids, one body presses on the other, touching it on some part of the surface. As a result, each of such bodies, in the absence of influence from the side of third bodies, accelerates or slows down. The interaction of both bodies is an act of action and reaction, which meets Newton's third law. If we consider as action the force of pressure on the first body from the side of the second, then the reaction will be the force developed by the first body and applied to the second body. The last force represents, as it were, the inertial resistance of the first body to a change in its speed. Thus, the reaction is manifested by the expression of the inertia property of the first body. This gave Newton an occasion to call this force the force of inertia. With the same acceleration caused by the first body, it is the greater, the greater its mass. But what is the mass? The concept of mass is defined both as a measure of the inertia m_{in} of the body, and as the amount of substance in a given body, which is considered equal to the gravitational mass m of the substance [1 - 3].

A. Einstein considered gravitational and inertial mass identical

$$m = m_{in}\,, \qquad (1)$$

and put the corresponding principle of equivalence at the heart of his theory of relativity [1]. In [4–6], other principles of equivalence were proposed and used, for comparison, which naturally yielded different results in terms of the mechanics of the interaction of hypothetical particles with negative and imaginary masses. The monograph [7] declared the principle of equivalence in the form of equality of the moduli of gravitational and inertial masses

$$|m| = |m_{in}|\,. \qquad (2)$$

The equivalence principle in the form (1) applies, first of all, to bodies with positive mass. Using it for the mechanics of negative particles [2] is a hypothetical extrapolation. The equivalence principle in the form (2) covers not only bodies with a negative one, but also, in various versions, with an imaginary and complex mass [4 - 6].

The aim of this work is to search for evidence of the equality of inertial and gravitational masses, if they are equal, and, in the case of their inequality, – quantitative dependence on each other.

2. The essence of the problem

They tried to solve the previously stated problem of equivalence of inertial and gravitational masses without taking into account the Mach principle [8–13], as if the interaction does not occur in the space of the global Universe, but independently, without the participation of third bodies, only between two interacting bodies.

The question is about the relationship between the three types of masses:

- m_a – active gravitational mass of the body, characterizing its ability to create a gravitational field;
- m_p − passive gravitational mass of a body, characterizing the impact of a gravitational field on it;
- m_{in} − inert mass is mass that characterizes the body's ability to acquire one or another acceleration under the influence of non-gravitational forces applied to it [9].

However, the last disclaimer that the occurrence of an inertia force requires only the action of forces of a non-gravitational nature on the body is a strange misconception: in order for the inertia forces to appear on the side of the body, as resistance forces to external forces, the forces acting on the body can be any nature, including gravitational. This is the conclusion that follows from the Mach principle. The posed question, according to [9], is actually a question of the ratio in which different interactions contribute to the inertial and gravitational masses of the body. For example, a study of the motion of elementary particles and ions in electromagnetic fields has shown that strong, weak, and electromagnetic interactions make equal contributions to an inert mass of a body. But it does not follow from anywhere that the field responsible for any interaction, ceteris paribus, cannot be a more intense or weaker source of the gravitational field than all other fields of matter. In this case, the active gravitational mass would not be equal to the inertial mass, and the difference would be the greater, the greater the fraction of the energy of this interaction in the total energy of the body. Therefore, in the general case, it cannot be ruled out that the active gravitational mass of the body will be different from its inert mass [9]:

$$m_a = m_{in} + \sum_A \eta_{a,A} \frac{E_A}{c^2}, \qquad (3)$$

where $\eta_{a,A}$ − the dimensionless parameter characterizing the nonequivalence of the contributions of the A-field to the active gravitational and inert mass of the body; E_A − the energy of the field responsible for the A-interaction of the components of the body. Similarly, a possible difference in the effect of an external gravitational field on different forms of matter can be described for a passive gravitational mass of a body:

$$m_p = m_{in} + \sum_A \eta_{p,A} \frac{E_A}{c^2}. \qquad (4)$$

Thus, only when the gravitational properties of different forms of matter will be the same. Otherwise, the universality of gravitational interaction will be violated.

To clarify the relationships between different masses of the same body, various experiments were repeatedly performed. These are, first of all, gravimetric measurements of the ratio of passive gravitational mass to inert mass. The general idea of these experiments was as follows.

Suppose that in expression (4) not everyone $\eta_{p,A}$ is equal to zero, as a result of which, in the general case, m_p it will not be equal m_i . Then the equations of motion of a point body in an external homogeneous gravitational field will take the form

$$m_i\mathbf{a} = \left(m_{in} + \sum_A \eta_{p,A}\frac{E_A}{c^2}\right)\mathbf{g} \qquad (5)$$

Where does that accelerations

$$\mathbf{a} = \left(1 + \sum_A \eta_{p,A}\frac{E_A}{m_i c^2}\right)\mathbf{g}, \qquad (6)$$

acquired by different bodies in the gravitational field will be different. Therefore, if two bodies having the same inertial mass but different composition are placed on the torsion pendulum rocker, then in the external gravitational field the pendulum will have a torque $\mathbf{M} = (m_{1p} - m_{2p})[\mathbf{rg}]$ proportional to the difference of the passive gravitational masses of these bodies, which is available for measurement with accuracy

$$\eta = (m_{1p} - m_{2p})/m_i. \qquad (7)$$

The most accurate measurements for the weak equivalence principle made in 2017 on the MICROSCOPE satellite [14] were determined with accuracy $|\eta| < 10^{-15}$. Although these results are perceived as evidence of the mutual equality of the passive and active gravitational masses, this does not mean that bodies of large sizes have gravitational and inert masses that coincide with the same accuracy. Since the gravimetric measurement of the ratio of the passive and gravitational masses of an extended body (planet) to its inert mass is impossible, effects should be sought in which a difference in these masses may manifest itself. Among these effects is the deviation of the motion of the center of mass of the extended body from the motion along the geodesic of the Riemannian space-time [9]. It was shown in [10] that in the case of an extended body in the field theory of gravity [11] and the relativistic theory of gravity [9, 12,] the passive gravitational mass of the Earth $m_{p\oplus}$ is not equal to the inert $m_{i\oplus}$

$$\frac{m_{p\oplus}}{m_{i\oplus}} = 1 + 7{,}6\cdot 10^{-10}\ . \qquad (8)$$

and for this reason, the center of mass does not move along the geodesic line, which can be observed in the experiment [10].

It was shown in [13, 15] that in the general theory of relativity (GR) the magnitude of the "inert mass" depends on the choice of a three-dimensional coordinate system, and therefore it has no physical meaning! For example, in one of the solutions, the dependence of the inertial mass on the gravitational mass is represented by the equation [15].

$$m_{in} = m(1 + \alpha^4) \qquad (9)$$

It follows that for the “inert mass" of the system consisting of matter and the gravitational field, in GR we can get, in view of the arbitrariness of the value, any given number in advance depending on the choice of spatial coordinates, although the gravitational mass of this system and all the effects of GR remain at this unchanged. With more complex transformations of spatial coordinates, leaving the metric asymptotically Galilean, the "inert mass" of the system can take any given values in advance, not only positive, which is significant, but also negative! It follows that statements like “principles of equivalence” about the equality of “inert” and “heavy” masses also have no physical meaning.

However, this equality takes place in a narrow class of three-dimensional coordinate systems, and since the "inert" and gravitational masses have different transformation laws, then when they switch to other three-dimensional coordinate systems, their equality is no longer fulfilled. In addition, the definition of "inert mass" in GR does not satisfy the principle of correspondence with Newton's theory. Due to the fact that the “inert mass” in GR depends on the choice of a three-dimensional coordinate system, its expression, in the general case, in an arbitrary three-dimensional coordinate system does not go over into the corresponding expression of Newton’s theory, because the “inert mass” in Newton’s theory does not depend on selection of spatial coordinates. Thus, in GR there is no classical Newtonian limit, and it does not satisfy the correspondence principle [15]. From the foregoing, a fundamentally important conclusion follows: if Einstein's theory is correct, then there is no “inert mass” in nature, since this concept itself is devoid of physical meaning.

3. The derivation of the basic formulas

The Mach principle [16, 17] consists in assigning the inertial motion of the body not to the Absolute Space (AS), but to a reference frame associated with distant stars, more precisely, to the center of mass of the Universe

$$\frac{d^2}{dt^2}\left[\frac{\sum m_i r_i}{\sum m_i}\right] = 0, \qquad (10)$$

where r_i — the distances from the moving body to the mass. Naturally, this automatically required an infinite speed of propagation of interactions. In itself, Mach's principle is obvious. In a deterministic universe, everything must influence everything. But Newton’s principle, which relates movement to AS, is also impeccable, since the existence of AS, if it does not have any effect on the material world, is equivalent to its non-existence. Therefore, it is natural that, formally, it is necessary to simultaneously take into account both of these principles, i.e. to determine the strength of the impact, take their additive amount. This is especially important if the AS turns out to be a substantial medium with resistance for the movement of bodies. Thus, Newtonian mechanics, ignoring the influence of distant stars, is to some extent approximate. The long-range principle, as a requirement of an infinite speed of propagation of interactions, does not contradict the short-range principle, according to which the speed of interaction does not exceed the speed of light. The impact of objects beyond the event horizon will be delayed, but they will occur. As a result, the distance between the body and the center of mass of the Universe will increase over time, the effective mass of the Universe, consisting of objects available to influence the body, will also increase, and in general, a slight corresponding drift of the final result will be observed over time.

Newton’s second law, also referred to as the basic law of dynamics [1], can be written [18]:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt}\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt} = \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{d\mathbf{v}}{dt} + \frac{d}{dt}\left(\frac{m_0}{\sqrt{1-v^2/c^2}}\right)\mathbf{v} =$$

$$= \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{d\mathbf{v}}{dt} + \frac{\mathbf{v}}{c^2}\frac{d}{dt}\frac{m_0 c^2}{\sqrt{1-v^2/c^2}}, \qquad (11)$$

where c — the speed of light, m — the mass of a body moving with speed v, $\mathbf{F} = \sum \mathbf{F}_k$ — the resultant of forces $\mathbf{F}_k$ acting on the body with a rest mass m_0, moving with speed $\mathbf{v}$. Obviously, the total speed of the body relative to all other bodies in the universe is zero. In this case, formula (11) will be simpler:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\boldsymbol{a} \qquad (12)$$

If the direction of action of the force $\mathbf{F} = F$ coincides with the direction of the acceleration of the motion of the body $\boldsymbol{a} = a$ arising under the action of this force, then formula (12) is still simplified:

$$F = ma \qquad (13)$$

In Newton's second law, mass plays the role of resistance to the action of force, so it is a measure of the inertia of the body, as a result of which formula (13) is written as:

$$F = m_{in}a, \qquad (14)$$

believing that this is a fundamentally different type of mass. For the gravitational interaction of two bodies with gravitational masses m_1 and m_2 , the law of universal gravitation also applies, which can be written

$$F = -\gamma\frac{m_1 . m_2}{r^2}, \qquad (15)$$

where r — the distance between interacting bodies, γ — the gravitational constant.

If we assume that a force F in the Absolute Space, in which there are no bodies, acts on a body A with a gravitational mass$m$, then this force will cause the acceleration of this body

$$a_1 = \frac{F}{m} \qquad (16)$$

The influence of the Universe on the body, in accordance with (15), is determined by the formula:

$$F_0 = -\gamma\frac{M \cdot m}{R^2} = ma_2, \qquad (17)$$

where M — the mass of the universe, R — the distance from the body A to the center of mass of the universe, whence

$$a_2 = -\gamma\frac{M}{R^2} \qquad (18)$$

$$a = a_1 + a_2 = \frac{F}{m} - \frac{\gamma\frac{M \cdot m}{R^2}}{m} \qquad (19)$$

$$ma = F - \gamma\frac{M \cdot m}{R^2} \qquad (20)$$

Since Newton's second law is written by formula (14), then

$$a = \frac{F}{m_{in}} \qquad (21)$$

Substituting (21) into (20), we obtain

$$\frac{m}{m_{in}}F = F - \gamma\frac{M \cdot m}{R^2} \qquad (22)$$

$$\frac{m}{m_{in}} = 1 - \gamma F \frac{M \cdot m}{R^2} \quad (23)$$

$$m_{in} = \frac{mF R^2}{F R^2 - \gamma M \cdot m} \quad (24)$$

$$m = \frac{m_{in} F R^2}{F R^2 + \gamma M \cdot m_{in}} \quad (25)$$

Formulas (24, 25) are the dependence of the inertial mass on the gravitational and gravitational mass on the inert, respectively. From their appearance it follows that these formulas, in addition to their dependence on the mass and size of the Universe, which should be, also depend on the magnitude of the force acting on the body.

Let us find the formula for Newton's second law taking into account the obtained dependence (24). To do this, substitute (24) in (14):

$$F = m_{in} a = ma \frac{F R^2}{F R^2 - \gamma M \cdot m} \quad (26)$$

$$F^2 R^2 - F\gamma M m = maF R^2 \quad (27)$$

$$FR^2 = maR^2 + \gamma M m \quad (28)$$

$$F = ma + \gamma \frac{M \cdot m}{R^2} = m\left(a + \gamma \frac{M}{R^2}\right) \quad (29)$$

4. Discussion of the results

Formulas (14, 24) indicate that if force F is applied to a body A with mass m, then law (14) will be satisfied. On the other hand, the basic law of dynamics is represented by formula (29), in which only gravitational mass is present, and there is no inert mass at all, which indicates the conditional character of the inert mass. Attempts to present the difference $m - m_{in}$ by some convenient formula for perception do not lead to acceptable results. The mass relations are easily representable in the form:

$$\frac{m}{m_{in}} = 1 - \gamma \frac{M \cdot m}{F R^2} \quad (30)$$

$$\frac{m_{in}}{m} = \frac{F R^2}{F R^2 - \gamma M \cdot m} \quad (31)$$

A feature of these dependencies is the presence of a force acting on the body and the gravitational mass of the body itself. The interdependence of inert and gravitational masses on the magnitude of the gravitational mass can be eliminated if we consider the reciprocal of the masses

$$\mu = \frac{1}{m} \quad (32)$$

$$\mu_{in} = \frac{1}{m_{in}} \quad (33)$$

For the reciprocal values of the masses, it is possible to write a formula for their difference, in which there is no gravitational mass on the right side:

$$\Delta = \mu - \mu_{in} = \gamma \frac{M}{FR^2}. \qquad (34)$$

Formula (34) hints that, possibly, inverse masses (32, 33) are more fundamental concepts for estimating the "amount of substance" in the body than ordinary mass. In particular, if in the Special Theory of Relativity, depending on the mass on the speed of the body, the mass is replaced by the reciprocal mass, then with an increase in the speed of the body to the speed of light, the reciprocal mass, as an indicator of the amount of substance in the body, will decrease to zero (unlike mass, which should grow to infinity, which is absurd).

This would mean that all the "substance" contained in the body passed into the kinetic energy of the body, i.e. into energy. And for this reason, there is no longer any possibility to accelerate the body to speeds greater than the speed of light, since the body itself has already been used up and it simply has not died. When conducting experiments to confirm the principles of equivalence of inert and gravitational masses [8, 9, 14, 19], the Mach principle was not taken into account as the influence of the gravitational influence of the Universe on a test body.

Recall that the Weak Equivalence Principle (WEP) states that the trajectory of an uncharged test body depends only on the starting point of its location and its initial speed and does not depend on its internal structure or composition [8]. The Einstein Principle of Equivalence (EPE) states that the WEP is fair and that the result of any non-gravity control experiment does not depend on the speed (freely falling) of the device, nor on where and when it is performed in the Universe [8]. The strong principle of equivalence states that the WEP is valid both for gravitating bodies and for any test bodies: the result of any gravitational or non-gravitational control experiment does not depend on the speed of a freely falling device and on where and when this experiment is carried out in the Universe [8].

Many experiments on testing equivalence principles are based on the idea [19] that for small fluctuations of a point mass on a weightless string of length l in the Earth's gravitational field with gravitational acceleration g, Newton's second law can be written as follows:

$$m_{in} \frac{d^2(l\alpha)}{dt^2} = -mg\alpha = -\gamma \frac{\alpha m \cdot M_z}{R_z^2} = F, \qquad (35)$$

where M_z and $R_z -$ the mass and radius of the Earth, respectively, $\alpha -$ the small deviation of the pendulum. The solution of equation (35) has the form of harmonic oscillations with a period

$$\tau_0 = 2\pi \sqrt{\frac{g}{l} \cdot \frac{m}{m_{in}}} \qquad (36)$$

Obviously, to check the principle of equivalence, it is necessary to make several pendulums of the same length and balls of different substances and measure the periods of their own vibrations. If the periods coincide, then the relations $\frac{m}{m_{in}}$ are the same. In this way, for example, in [19] using aluminum and platinum, the equality of inertial and gravitational masses with a relative error $\delta = \pm 0{,}9 \cdot 10^{-12}\ \%$ was established.

However, the original equation (35) is not accurate enough; it does not take into account the influence of the entire part of the Universe remaining outside the Earth. The relative error δ_0 of this method due to the use of Newton's second law (14) under the assumption of the action of EPE (1), instead of using formula (29), we find from the ratio:

$$ma + \gamma \frac{M \cdot m}{R^2} \quad - \quad 100\ \%$$

$$\gamma \frac{M \cdot m}{R^2} \quad - \quad \delta_0 \ \%$$

$$\delta_0 = \frac{100}{\frac{aR^2}{\gamma M}+1} \ \% \qquad (37)$$

Using the known data, $\gamma = 6{,}67259 \cdot 10^{-11} \ \frac{m^3}{kg \cdot s^2}$, $M \approx 10^{52} \ kg$, $R \approx 10^{27} \ m$ [20, 21], it is easy to verify that for accelerations $a > 10^{-10} \ \frac{m}{s^2}$, the unit in the denominator in formula (37) can be omitted, then we get:

$$\delta_0 \approx \frac{100\gamma M}{aR^2} \ \% \qquad (38)$$

In particular, for acceleration $a = g = 9{,}8 \ \frac{m}{s^2}$, i.e. for the experimental conditions [19], we obtain

$$\delta_0 \approx 6{,}67 \cdot 10^{-12} \ \% \qquad (39)$$

Comparing the values δ and δ_0, we see that the error from the use of Newton's second law without taking into account the response of the Universe is greater than from other reasons that are possible during this experiment.

In order to take into account the gravitational influence of the Universe, it is necessary m_{in} in equation (35), replace it with its value from formula (24), and instead of the acting force F included in formula (24), substitute its expression through the law of universal gravitation, written on the right side of the equation (35). Naturally, after such transformations, the new equation will be much more complicated.

But he will have an important advantage: the solution of this equation will be based on the interdependence of inert and gravitational masses that takes place in reality.

Conclusions

1. The dependences of inertial and gravimetric masses from each other are derived taking into account the response of the Universe to interacting bodies.

2. It was found that the inertial mass depends not only on the magnitude of the corresponding gravitational mass, but also on the force acting on the body.

3. Formulas are found for the relations of inert and gravitational masses with the condition that the Mach principle is fulfilled.

4. It is shown that the relations of inert and gravitational masses depend not only on the force acting on the body, but also on the gravitational mass of this body.

5. It has been revealed that if instead of the ratio of inert and gravitational masses we choose the difference of the inverse values of these masses, the resulting formula turns out to be independent of the gravitational mass of the body.

6. The basic law of dynamics is agreed taking into account the response of the Universe to interacting bodies in accordance with the Mach principle.

7. It is shown that the error from using Newton's second law without taking into account the response of the Universe to interacting bodies in many experiments can exceed the error caused by other factors.

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