Analyzing q-Laguerre Polynomials: The Recurrence Relation Perspective

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Introduction

The study of q-Laguerre polynomials encompasses a rich area of mathematical research, delving into the realm of special functions, orthogonal polynomials, and q-series. These polynomials, introduced as q-analogs of classical Laguerre polynomials, play a significant role in various branches of mathematics, including combinatorics, quantum mechanics, and statistical mechanics. One fruitful approach for analyzing q-Laguerre polynomials involves the use of recurrence relations to derive expansion coefficients and connection coefficients within series representations, offering insights into their properties and applications.

Description

The q-Laguerre polynomials, denoted as L_n(x;q), are a family of orthogonal polynomials defined on the interval [0,∞) with respect to the weight function w(x;q) = x^α * exp_q(-x), where α>-1 and exp_q(-x) denotes the q-exponential function. These polynomials arise in the study of special functions associated with quantum groups, q-series identities, and combinatorial problems involving partitions and compositions. The recurrence relation approach for expansion and connection coefficients in series of q-Laguerre polynomials leverages the properties of orthogonal polynomials and q-analog techniques to derive explicit formulas and relationships. One fundamental recurrence relation for q-Laguerre polynomials is the three-term recurrence relation, which relates consecutive polynomials in the sequence and serves as a building block for deriving expansion coefficients. By applying the three-term recurrence relation to series representations of q-Laguerre polynomials, researchers can establish connections between different polynomial coefficients, facilitating the computation of expansion coefficients and their properties. This approach involves manipulating q-series identities, generating functions, and special function properties to derive closed-form expressions for expansion coefficients in terms of q-binomial coefficients, q-factorials, and other q-analogues. The recurrence relation approach also extends to connection coefficients between q-Laguerre polynomials and other orthogonal polynomials, such as q-Hermite polynomials or q-Jacobi polynomials. These connection coefficients play a crucial role in transforming between different orthogonal polynomial bases, providing insights into the relationship between different q-special functions and their underlying structures. Moreover, the recurrence relation approach allows for the exploration of properties such as orthogonality, symmetry, and asymptotic behavior of q-Laguerre polynomials. By analyzing the recurrence relations and their consequences, researchers can uncover hidden symmetries, recurrence patterns, and generating functions associated with q-Laguerre polynomials, contributing to a deeper understanding of their mathematical properties. The applications of the recurrence relation approach for expansion and connection coefficients in series of q-Laguerre polynomials extend to various mathematical and scientific fields. In quantum mechanics, for instance, q-Laguerre polynomials arise in the context of quantum harmonic oscillators with q-deformed algebraic structures, providing solutions to energy eigenvalue problems and wave functions. In statistical mechanics and probability theory, q-Laguerre polynomials play a role in modeling random processes, generating probability distributions, and solving difference equations with q-analogues. Their connection to q-series identities and combinatorial methods also makes them valuable tools in combinatorial enumeration, partition theory, and discrete mathematics.

Conclusion

The recurrence relation approach for expansion and connection coefficients in series of q-Laguerre polynomials offers a systematic and insightful method for analyzing these special functions. By leveraging recurrence relations, q-analog techniques, and orthogonal polynomial properties, researchers gain a deeper understanding of the mathematical structure, properties, and applications of q-Laguerre polynomials across various mathematical and scientific disciplines.