Deformation Due to Mechanical & Electromagnetic Forces in A Magneto-Micropolar Plate Irradiated by Thermal Pulsed Laser

[1]Arvind Kumar, [2]Devinder Singh

[1]Department of Mathematics, I.K.G. Punjab Technical University Jalandhar

[1]Department of Mathematics, GNDEC Ludhiana, Punjab (India)

arvi.math@gmail.com

Abstract

The purpose of this paper is to study the elastodynamical interactions in magneto-micropolar thermoelastic half-space considering the effect of hall current, laser heat source and rotation subjected to input ultra-laser heat source. The micropolar theory of thermoelasticity by Eringen (1966) has been used to investigate the problem. Normal mode analysis technique has been used to solve the resulting non–dimensional coupled field equations to obtain displacement, stress components and temperature distribution. Numerical computed results of all the considered variables have been shown graphically to depict the combined effect of hall current, laser heat source and rotation on the phenomena. Some particular cases of interest are also deduced from the present study.

Keywords: Micropolar thermoelastic, Hall Current, Thermal laser heat source, rotation.

Introduction:

The problems involving the investigation of effect of magnetic field (that may be earth's magnetic field or other human generated high intensity magnetic field) and thermal loading by lasers on various type of materials are of great importance in seismological research and in engineering applications. The linear theory of micropolar elasticity was developed by Eringen [1]. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Rigid chopped fibers, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals are examples of such materials.

The theory of magneto thermoelasticity has a wide range of applications and possibilities of research in the field of geology, earth sciences, plasma physics and engineering. When a particle is stationary under the effect of magnetic field, the field has no effect on this particle. Also, consider a particle is moving in parallel direction of the magnetic field, the particle will move undeflected. Now in case a particle is moving in path having a component normal to magnetic field, the particle will be deflected due to a force acting on it. In addition to this deflected motion this particle will experiences the electric field. The combined force is Lorentz force. There is a consideration that mechanical and electromagnetic fields interactions take place due to Lorentz forces. Conductivity perpendicular to the direction of magnetic field is decreased due to the free spiraling of negatively charged electrons and other ions about the magnetic field lines before colliding and a current is induced perpendicular to electric field and magnetic fields both. This phenomenon is called the Hall Effect. When the magnetic field intensity is very high Hall Effect cannot be neglected. Zakaria [2] investigated the effects of Hall current and rotation on magneto micropolar generalized thermoelasticity including the boundary condition with a source of ramp type heating.

A thermal shock induces very rapid movement in the structural elements, giving the rise to very significant inertial forces, and give rise to oscillations. The ultra-short lasers have pulse durations ranging from nanoseconds to femto seconds. Also, in ultra-short laser pulse, the high energy flux and short duration result in a very large thermal gradient. So, Fourier law of heating is no longer valid. Scruby et al. [3] investigated a mathematical model of point source to study the ultrasonic evolution by lasers. He studied the physics of heated plate by laser heat loading in the thermoelastic system as a surface center of expansion (SCOE). Also, for one-point laser heat input Rose [4] provided more accurate mathematical basis. Later McDonald [5] and Spicer [6] gave a mathematical model known as laser-generated ultrasound model by introducing thermo-diffusion concept. Dubois [7] verified by experimental results that penetration depth plays an important role in the generation of laser-ultrasound. Abo-Dahab and Abbas [8] investigated LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Chen et al. [9] and Kim et al. [10] investigated some other such type of research. Thermoelastic behavior of laser heat in context of different theories of thermoelasticity was presented by Youssef and Al-Bary [11]. A 2- dimensional problem in generalized thermoelastic medium with thermo-diffusion was investigated by Elhagary [12]. Kumar et al. [13] studied the elastodynamical interactions of input heat source with microstretch thermoelastic medium. The aim of the present study is to investigate the interaction in magneto micropolar thermoelastic medium, taking into consideration the effect of hall current, laser heat source and rotation. The components of displacement, stress, current density and temperature distribution are obtained by using normal mode analysis. The problem has become more interesting with the inclusion of thermal laser heat source, normal and tangential forces. The resulting quantities are computed numerically and depicted graphically.

Basic equations

Let us consider a micropolar thermoelastic medium permeated by an initial strong magnetic field $\boldsymbol{H} = (0, H_0, 0)$ and the considered medium is rotating. The angular velocity is assumed to be equal to Ω. For magneto-micropolar thermoelastic medium the basic equations and constitutive relation in absence of body forces, body couples and stretch forces, following Eringen [1], Al Qahtani and Dutta [14] and Zakaria [2] are given by:

$$(\lambda + \mu)\nabla(\nabla.\boldsymbol{u}) + (\mu + K)\nabla^2\boldsymbol{u} + K\nabla \times \boldsymbol{\phi} - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla T + \mu_0\epsilon_{rji}J_rH_j = \rho\left(\ddot{\boldsymbol{u}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u}) + 2\boldsymbol{\Omega} \times \frac{\partial \boldsymbol{u}}{\partial t}\right), \quad (1)$$

$$(\gamma\nabla^2 - 2K)\boldsymbol{\phi} + (\alpha + \beta)\nabla(\nabla.\boldsymbol{\phi}) + K\nabla \times \boldsymbol{u} = \rho j\left(\ddot{\boldsymbol{\phi}} + \boldsymbol{\Omega} \times \frac{\partial \boldsymbol{\phi}}{\partial t}\right), \quad (2)$$

$$K^*\nabla^2 T = \rho c^*\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \left(1 + \varepsilon\tau_0\frac{\partial}{\partial t}\right)\beta_1 T_0(\nabla.\dot{\boldsymbol{u}} - Q) + \nu_1 T_0\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*, \quad (3)$$

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijk}\phi_k\right) - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T, \quad (4)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \quad (5)$$

where $\lambda,\ \mu,\ \alpha,\ \beta,\ \gamma, and\ K\ are$ constants depending on the nature of material, ρ is density of the medium, $\boldsymbol{u} = (u_1, u_2, u_3)$ and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ are displacement and microrotation vectors respectively, T is temperature, T_0 is the reference temperature, K^* is the coefficient of the thermal conductivity, c^* is the specific heat at constant strain, j is the microinertia, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$,$\nu_1 = (3\lambda + 2\mu + K)\alpha_{t2}$, $\alpha_{t1}\ and\ \alpha_{t2}$ are coefficients of linear thermal expansion, t_{ij} are components of stress, m_{ij} are components of couple stress, δ_{ij} is Kroneker delta function, $\tau_0\ and\ \tau_1$ are thermal relaxation times with $\tau_0 \geq \tau_1 \geq 0$.

Let the microstretch thermoelastic medium is rotating with angular velocity $\boldsymbol{\Omega}$. The equations of motion have two extra terms,

(i) The centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u})$ due to time varying motion.
(ii) The Coriolis acceleration $2(\boldsymbol{\Omega} \times \dot{\mathbf{u}})$.

The current density vector $\boldsymbol{J}$ can be expressed as:

$$\boldsymbol{J} = \frac{\sigma_0}{1+m^2}\left[\boldsymbol{E} + \mu_0(\dot{\boldsymbol{u}} \times \boldsymbol{H}) - \frac{\mu_0}{e n_e}(\boldsymbol{J} \times \boldsymbol{H})\right]. \quad (6)$$

Here $\boldsymbol{F} = \mu_0(\boldsymbol{J} \times \boldsymbol{H})$ is the Lorentz force, $\boldsymbol{H}$ is the magnetic field vector, $\boldsymbol{E}$ is the intensity of electric field, m is the Hall parameter, σ_0 is the electrical conductivity, e is the charge of an electron, n_e is the number density of electrons. Further the plate surface is illuminated by laser pulse given by the heat input

$$Q = I_0 f(t) g(x_1) h(x_3), \tag{7}$$

$$f(t) = \frac{t}{t_0^2} e^{-\left(\frac{t}{t_0}\right)}, \tag{8}$$

$$g(x_1) = \frac{1}{2\pi r^2} e^{-\left(\frac{x_1^2}{r^2}\right)}, \tag{9}$$

$$h(x_3) = \gamma^* e^{-\gamma^* x_3} \tag{10}$$

$where$, I_0 energy absorbed, t_0 is the pulse rising time,r is the beam radius.
Equation (7) with substitution of (8- 10) takes the form

$$Q = \frac{I_0 \gamma^*}{2\pi r^2 t_0^2} t e^{-\left(\frac{t}{t_0}\right)} e^{-\left(\frac{x_1^2}{r^2}\right)} e^{-\gamma^* x_3} \tag{11}$$

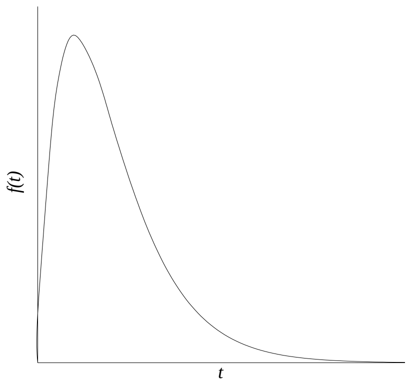

Fig. 1. Temporal profile of $f(t)$

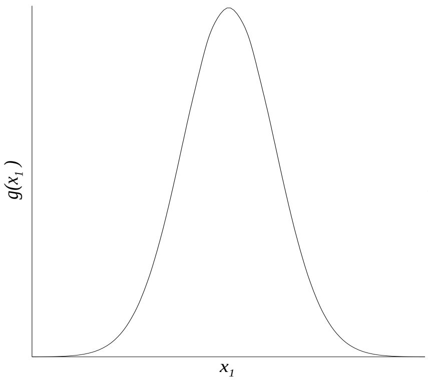

Fig. 2. Profile of $g(x_1)$

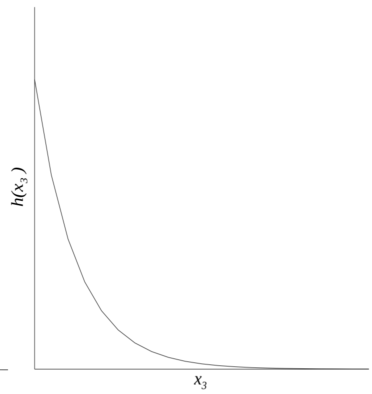

Fig. 3. Profile of $h(x_3)$

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" $\cdot$ ") denotes the derivative with respect to time respectively.

Formulation of the problem:

We consider a magneto-micropolar thermoelastic medium with rectangular Cartesian coordinate system $OX_1X_2X_3$ having x_3-axis pointing vertically downward the medium. A normal force/tangential force and ultra-short laser pulse are assumed to acting at the origin of the rectangular Cartesian co-ordinate system. A component of Hall current H_0 is in x_2-direction.

We consider plane strain problem with all the field variables depending on (x_1, x_3, t). For two dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \tag{12}$$

For further consideration, it is convenient to introduce in equations (1)-(3) the dimensionless quantities defined as:

$$x_i' = \frac{\omega^*}{c_1} x_i\,,\ u_i' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_i, \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0} \phi_i\,, T' = \frac{T}{T_0}\,, t' = \omega^* t\,, \tau_1' = \omega^* \tau_1\,, \tau_0' = \omega^* \tau_0\,, t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij}\,,$$

$$\omega^* = \frac{\rho c^* c_1^2}{K^*}\,,\ c_1^2 = \frac{\lambda + 2\mu + k}{\rho}, \quad m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij}\,, \quad \Omega^1 = \frac{\Omega}{\omega^{*\prime}}, \quad M = \frac{\sigma_0 \mu_0^2 H_0^2}{\rho\omega^*}\,, \quad Q' = \frac{\beta_1^2}{\rho c_1^2} Q \tag{13}$$

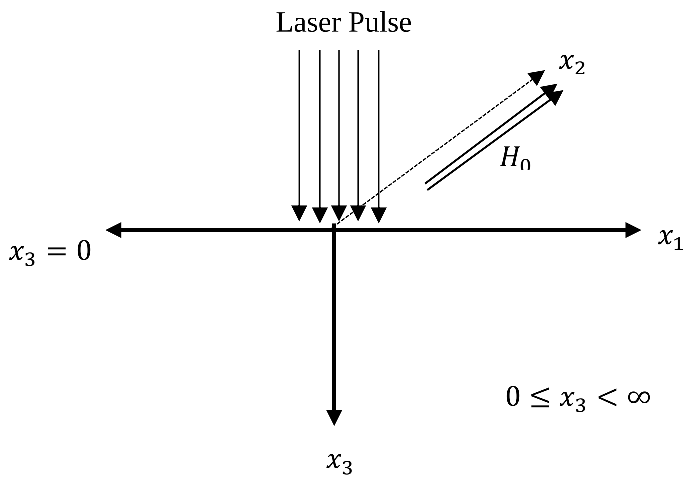

Fig. 4. Geometry of the problem.

Making use of equation (12)-(13) the system of equations (1)-(3) reduces to:

$$\zeta_1 \frac{\partial e}{\partial x_1} + \zeta_2 \nabla^2 u_1 - \zeta_3 \frac{\partial \phi_2}{\partial x_3} + {\Omega_0}^2 u_1 - 2\Omega_0 \frac{\partial u_3}{\partial t} + \frac{M}{1+m^2}\left(\frac{\partial u_1}{\partial t} + m\frac{\partial u_3}{\partial t}\right) - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \ddot{u}_1, \tag{14}$$

$$\zeta_1 \frac{\partial e}{\partial x_3} + \zeta_2 \nabla^2 u_3 + \zeta_3 \frac{\partial \phi_2}{\partial x_1} + 2\Omega_0 \frac{\partial u_1}{\partial t} + {\Omega_0}^2 u_3 - \frac{M}{1+m^2}\left(m\frac{\partial u_1}{\partial t} - \frac{\partial u_3}{\partial t}\right) - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \ddot{u}_3, \tag{15}$$

$$\nabla^2 \phi_2 - 2\zeta_4 \phi_2 + \zeta_4 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = \zeta_5 \ddot{\phi}_2, \tag{16}$$

$$-\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \zeta_6 \left(1 + \varepsilon \tau_0 \frac{\partial}{\partial t}\right)(\dot{e} - Q) = Q_0 f^*(x_1, t) e^{-\gamma^* x_3}, \tag{17}$$

$$M_1 = 2\Omega_0 + \frac{M}{1+m^2}, \quad \zeta_1 = \frac{\lambda + \mu}{\rho c_1^2}, \zeta_2 = \frac{\mu + K}{\rho c_1^2}, \zeta_3 = \frac{K}{\rho c_1^2}, \zeta_4 = \frac{K c_1^2}{\gamma \omega^{*2}}, \zeta_5 = \frac{\rho j c_1^2}{\gamma}, \zeta_6 = \frac{\beta_1 T_0^2}{\rho \omega^* K^*}$$

Using Helmholtz's theorem the displacement components u_1 and u_3 are related to the non- dimensional potential functions ϕ and ψ by the relation mentioned below:

$$u_1 = \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} - \frac{\partial \psi}{\partial x_1} \tag{18}$$

Substituting the values of u_1 and u_3 from (18) in (14)-(17), we obtain:

$$\left(\nabla^2 + \Omega^2 + \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\phi + \left(2\Omega - \frac{mM}{1+m^2}\right)\frac{\partial \psi}{\partial t} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = 0, \tag{19}$$

$$\left(a_3 \nabla^2 + \frac{M}{1+m^2}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right)\psi - \left(2\Omega - \frac{mM}{1+m^2}\right)\frac{\partial \phi}{\partial t} - a_4 \phi_2 = 0, \tag{20}$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\dot{T} + a_5 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right)\nabla^2 \phi - \nabla^2 T = Q_0 f^*(x_1, t) e^{-\gamma^* x_3}, \tag{21}$$

$$\left(\nabla^2 - 2a_1 + a_2 \frac{\partial^2}{\partial t^2}\right)\phi_2 + a_1 \nabla^2 \psi = 0 \tag{22}$$

where

$a_1 = \frac{Kc_1^2}{\gamma\omega^{*2}}$, $a_2 = -\frac{\rho j c_1^2}{\gamma}$, $a_3 = \frac{\mu+K}{\rho c_1^2}$, $a_4 = \frac{K}{\rho c_1^2}$, $a_5 = \frac{\beta_1^2 T_0}{\rho K^* \omega^*}$, $a_6 = \frac{\lambda+\mu}{\rho c_1^2}$, $Q_0 = \frac{a_{13} I_0 \gamma^*}{2\pi r^2 t_0^2}$,

$f(x_1, t) = \left[t + \varepsilon\tau_0\left(1 - \frac{t}{t_0}\right)\right] e^{-\left(\frac{x_1^2}{r^2} + \frac{t}{t_0}\right)}$.

Solution of the problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi, \psi, T, \phi^*\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{T}, \overline{\phi^*}\}(x_3) e^{i(kx_1 - \omega t)}. \qquad (23)$$

Here ω is the angular frequency and k is wave number.

Making use of (23) in equations (19)-(22) and after some simplifications, yield:

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\phi} = f_1(\gamma^*, x_1, t) e^{-\gamma^* x_3}, \qquad (24)$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{T} = f_2(\gamma^*, x_1, t) e^{-\gamma^* x_3}, \qquad (25)$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\phi}_2 = f_3(\gamma^*, x_1, t) e^{-\gamma^* x_3}, \qquad (26)$$

$$[A\boldsymbol{D}^8 + B\boldsymbol{D}^6 + C\boldsymbol{D}^4 + E\boldsymbol{D}^2 + F]\bar{\psi} = f_4(\gamma^*, x_1, t) e^{-\gamma^* x_3}. \qquad (27)$$

Here, $\boldsymbol{D} = \frac{d}{dx_3}$

$k_1 = -k^2 + \Omega^2 - \frac{i\omega M}{1+m^2} + \omega^2, k_2 = -i\omega\left[2\Omega - \frac{mM}{1+m^2}\right], k_3 = \omega^2 + \Omega^2 - i\omega\frac{M}{1+m^2} - a_3 k^2,$

$k_4 = k^2 + \omega^2 a_2 + 2a_1, k_5 = i\omega(1 - i\omega\tau_0) - k^2, k_6 = a_5(i\omega + \omega^2\varepsilon\tau_0), k_7 = a_3(k_4 + k_5) + k_3 + a_3 k_4$

$k_8 = a_1 a_4 k^2 - k_3 k_4 - k_5(k_3 - a_1 a_4 + a_3 k_4), k_9 = k_5(a_1 a_4 k^2 - k_3 k_4), k_{12} = k_6 k^2(k_3 k_4 - a_1 a_4 k^2),$

$k_{10} = k_6(a_3 k^2 + k_3 + a_3 k_4 - a_1 a_4), k_{11} = k_6(2a_1 a_4 k^2 - k_3 k_4 - a_3 k_4 k^2 - k_3 k^2), A = -a_3,$

$B = k_7 - a_3 k_1 - \tau_{11} a_3 k_6, C = k_8 + k_1 k_7 + \tau_{11} k_{10} - k_2^2, E = k_1 k_8 - k_9 + \tau_{11} k_{11} + k_2^2(k_4 + k_5),$

$F = \tau_{11} k_{12} - k_1 k_9 - k_2^2 k_4 k_5.$

The solution of the above system of equations (24)-(27) satisfying the radiation conditions that $(\bar{\phi}, \bar{\psi}, \bar{T}, \overline{\phi_2}) \to 0$ as $x_3 \to \infty$are given as following:

$$\bar{\phi} = \sum_{i=1}^{4} c_i e^{-m_i x_3} + \frac{f_1}{f_5} e^{-\gamma^* x_3}, \qquad (28)$$

$$\bar{T} = \sum_{i=1}^{4} \alpha_i c_i e^{-m_i x_3} + \frac{f_2}{f_5} e^{-\gamma^* x_3}, \qquad (29)$$

$$\overline{\phi_2} = \sum_{i=1}^{4} \beta_i c_i e^{-m_i x_3} + \frac{f_3}{f_5} e^{-\gamma^* x_3}, \tag{30}$$

$$\bar{\psi} = \sum_{i=1}^{4} \delta_i c_i e^{-m_i x_3} + \frac{f_4}{f_5} e^{-\gamma^* x_3}. \tag{31}$$

Here$m_i^2 (i = 1,2,3,4)$ are the roots of characteristic equation of equation (24).

$$\alpha_i = -\frac{-k_6 a_3 m_i^6 + k_{10} m_i^4 + k_{11} m_i^2 + k_{12}}{-a_3 m_i^6 + k_7 m_i^4 + k_8 m_i^2 - k_9},\ \beta_i = \frac{a_1 k_2 m_i^4 - a_1 k_2 (k_5 + k^2) m_i^2 + a_1 k_2 k_5 k^2}{-a_3 m_i^6 + k_7 m_i^4 + k_8 m_i^2 - k_9},$$

$$\delta_i = \frac{k_2 m_i^4 - k_2 (k_5 + k_4) m_i^2 + k_2 k_4 k_5}{-a_3 m_i^6 + k_7 m_i^4 + k_8 m_i^2 - k_9}, \quad i = 1,2,3,4,$$

$$f_1 = Q_0 f(x_1, t)\left(-a_3 \gamma^{*6} + k_7 \gamma^{*4} + k_8 \gamma^{*2} - k_9\right),$$

$$f_2 = Q_0 f(x_1, t)\left(-k_6 a_3 \gamma^{*6} + k_{10} \gamma^{*4} + k_{11} \gamma^{*2} + k_{12}\right),$$

$$f_3 = Q_0 f(x_1, t)\left(a_1 k_2 \gamma^{*4} - a_1 k_2 (k_5 + k^2) \gamma^{*2} + a_1 k_2 k_5 k^2\right),$$

$$f_4 = Q_0 f(x_1, t)\left(k_2 \gamma^{*4} - k_2 (k_5 + k_4) \gamma^{*2} + k_2 k_4 k_5\right),$$

$$f_5 = \left(A \gamma^{*8} + B \gamma^{*6} + C \gamma^{*4} + E \gamma^{*2} + F\right).$$

Substituting the values of $\bar{\phi}, \bar{T}, \bar{\phi}_2, \bar{\psi}$ from the equations (28)-(31) in the (4)-(5), and using (12)-(13), (18) and solving the resulting equations, we obtain:

$$\bar{t}_{33} = \sum_{i=1}^{4} G_{1i} e^{-m_i x_3} - M_1 e^{-\gamma^* x_3}, \tag{32}$$
$$\bar{t}_{31} = \sum_{i=1}^{4} G_{2i} e^{-m_i x_3} - M_2 e^{-\gamma^* x_3}, \tag{33}$$
$$\bar{m}_{32} = \sum_{i=1}^{4} G_{3i} e^{-m_i x_3} - M_3 e^{-\gamma^* x_3}, \tag{34}$$
$$\bar{T} = \sum_{i=1}^{4} G_{4i} e^{-m_i x_3} - M_4 e^{-\gamma^* x_3} \tag{35}$$

$G_{mi} = g_{mi} C_i$, $i = 1,2, \dots, 4$. g_{mi} and M_i are mentioned in appendix A.

Boundary Conditions:

We consider normal and tangential forces acting at the surface $x_3 = 0$ along with vanishing of couple stress at $x_3 = 0$ and $I_0 = 0$. Mathematically this can be written as:

$$t_{33} = -F_1 e^{-(kx_1 - \omega t)},\ t_{31} = -F_2 e^{-(kx_1 - \omega t)}, m_{32} = 0, \frac{\partial T}{\partial x_3} = 0 \tag{36}$$

where F_1 and F_2 are the magnitude of the applied forces.

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following equations:

$$\sum_{i=1}^{4} (g_{1i}, g_{2i}, g_{3i}, g_{4i}) c_i = (-F_1, -F_2, 0, 0). \tag{37}$$

The system of equations (37) are solved by using the matrix method as follows:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}^{-1} \begin{bmatrix} -F_1 \\ -F_2 \\ 0 \\ 0 \end{bmatrix}. \quad (38)$$

Special cases:

Micropolar Thermoelastic Solid

If we neglect the Hall current in Equations (37) and put $I_0 = 0$, we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic half space.

Numerical Results and Discussions:

The analysis is conducted for a magneto-micropolar material. For numerical computations, following Eringen [15], the values of physical constants are:

$\lambda = 9.4 \times 10^{10}\mathrm{Nm^{-2}}, \mu = 4.0 \times 10^{10}\mathrm{Nm^{-2}}, K = 1.0 \times 10^{16}\mathrm{Nm^{-2}}, \rho = 1.74 \times 10^{3}\mathrm{Kgm^{-3}}, j = 0.2 \times 10^{-19}\mathrm{m^2}, \quad \gamma = 0.779 \times 10^{-9}\mathrm{N}$,

Following Dhaliwal [16] thermal parameters are given by:

$c^* = 1.04 \times 10^{3}\mathrm{JKg^{-1}K^{-1}},\ K^* = 1.7 \times 10^{6}\mathrm{Jm^{-1}s^{-1}K^{-1}},\ \alpha_{t1} = 2.33 \times 10^{-5}\mathrm{K^{-1}}, \alpha_{c1} = 2.48 \times 10^{10}\mathrm{K^{-1}},\ \ T_0 = 298\mathrm{K},\ \ \tau_0 = 0.02,\ \tau_1 = 0.01, \alpha_{c1} = 2.65 \times 10^{-4}\mathrm{m^3Kg^{-1}},\ a = 2.9 \times 10^{4}\mathrm{m^2s^{-2}K^{-1}}, b = 32 \times 10^{5}\mathrm{Kg^{-1}m^5s^{-2}}, \tau^1 = 0.04,\ \tau^0 = 0.03,\ D = 0.85 \times 10^{-8}\mathrm{Kgm^{-3}s}$

A comparison of the dimensionless form of the field variables for the cases of micropolar thermoelastic with Hall current, rotation and input laser heat source (MPHCLSR) and micropolar thermoelastic (MPTH) is presented in Figures 4-9. The values of all physical quantities for all cases are shown in the range $0 \le x_3 \le 5$.

Solid lines, dash lines corresponds to micropolar thermoelastic with Hall current and input laser heat source (MPHCLSR) and micropolar thermoelastic (MPTH), respectively for $t = 0.1$

The computations were carried out in the absence and presence of laser pulse ($I_0 = 10^5 \& I_0 = 0$) and on the surface of plane $x_1 = 1, t = 0.1$.

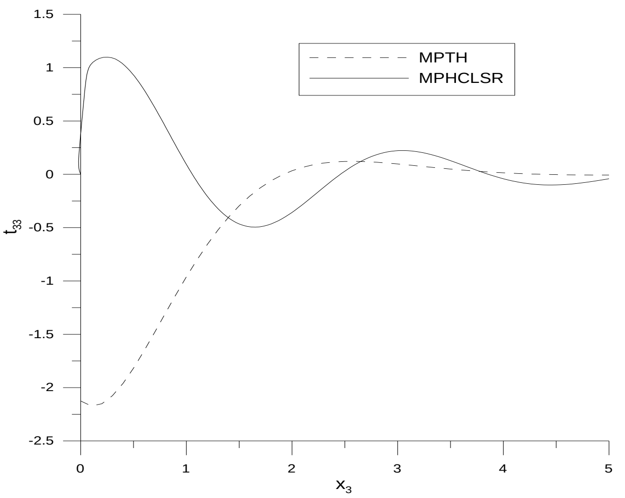

Fig. 5. Variation of t_{33} w.r.t. x_3

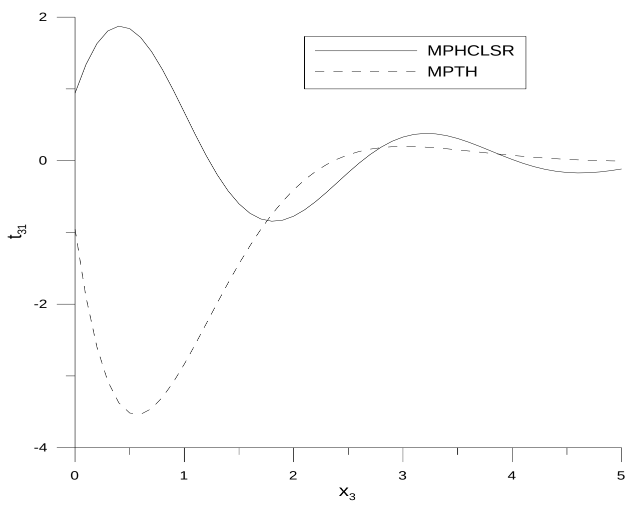

Fig. 6. Variation of t_{31} w.r.t. x_3

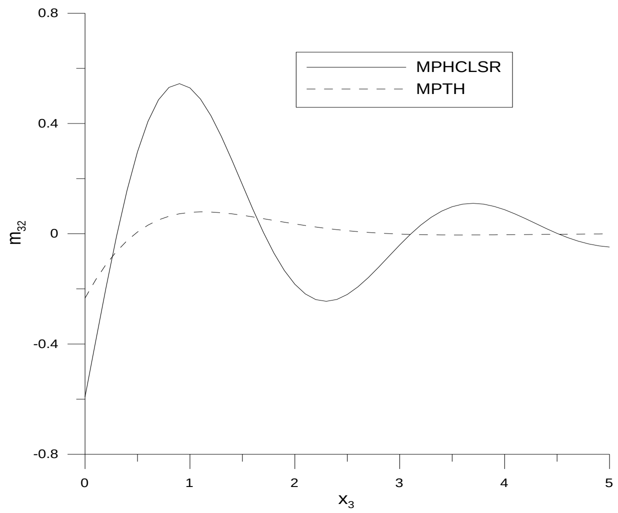

Fig. 7. Variation of m_{32} w.r.t. x_3

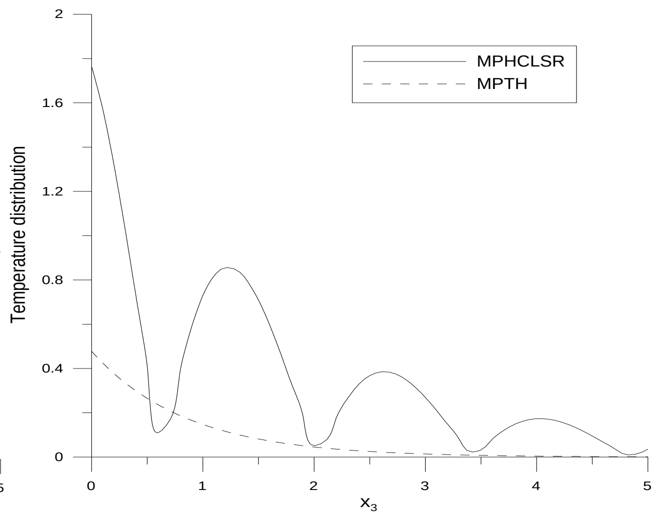

Fig. 8. Variation of temperature w.r.t. x_3

Fig. 5 shows the variation of normal stress t_{33} with the distance x_3. It is noticed that for MPHCLSR and MPTH, the normal stress t_{33} show opposite behavior initially. The normal stress in MPHCLSR initially increases and then show oscillatory trend. The value of t_{33} approaches to boundary surface away from the source.

Fig. 6 displays the variation of tangential stress t_{31} with the distancex_3. It is noticed that initially the behavior of t_{31} for MPHCLSR and MPTH is opposite. Initially t_{31} increases monotonically for MPHCLSR and decreases monotonically for MPTH but approaches to the boundary surface away from the point of application of normal force.

Fig. 7 clears the variation of couple stress m_{32} with distance x_3 for MPHCLSR and MPTH. The variation of m_{32} for (MPHCLSR and MPTH) is monotonically increasing in the region $0 \leq x_3 \leq 1$ and monotonically decreasing thereafter. The m_{32} approaches to zero away from the point of application of source. It is clear from figure 3 that Hall current has a significant effect on the value ofm_{32} and causes significant oscillatory behavior in MPHCLSR.

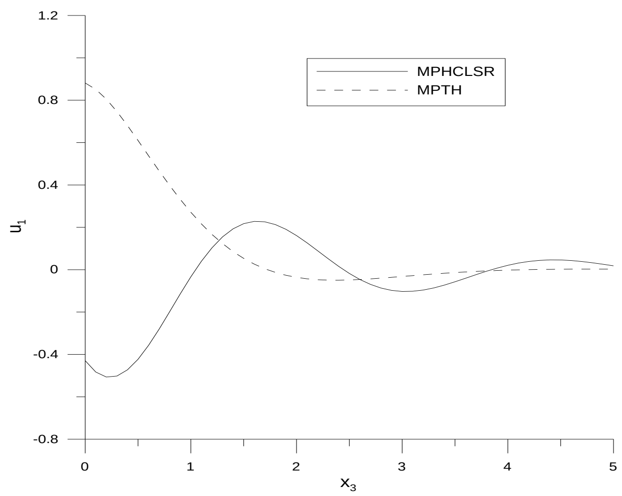

Fig. 9. Variation of u_1 w.r.t. x_3

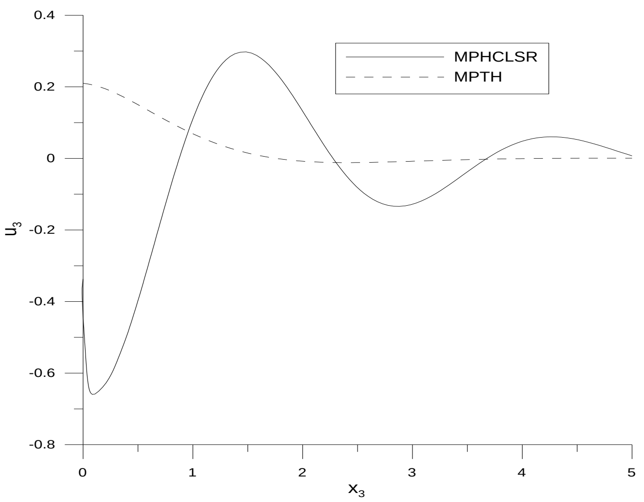

Fig. 10. Variation of u_3 w.r.t. x_3

Fig. 8 displays the variation of temperature T with distance x_3. The values of temperature change for MPTH show monotonically decreasing behavior in the range $0 \leq x_3 \leq 5$. In case of MPHCLSR the temperature decreases by exhibiting oscillatory trend due to the Hall Effect and input laser heat source.

Fig. 9 and Fig. 10 exhibit the behavior of displacement components u_1 and u_3 w.r.t. x_3. Both the displacement components approach to boundary surface away from the application of normal force which is in agreement to the generalized theory of thermoelasticity.

Conclusions:

The problem consists of investigating displacement components, temperature distribution, Hall current and stress components in a homogeneous isotropic micropolar thermoelastic half space due to various sources subjected to laser pulse. Normal mode analysis technique is employed to express the results mathematically. The analysis of results permits some concluding remarks:

(1) It is clear from the figures that all the field variables have nonzero values only in the bounded region of space indicating that all the results are in agreement with the generalized theory of thermoelasticity.

(2) The effect of the Hall current, rotation and ultra-laser is much pronounced in all the resulting quantities.

The new model is employed in magneto-micropolar thermoelastic medium as a new improvement in the field of thermoelasticity. The subject becomes more interesting due to Hall current involving rotation and irradiation of an ultra-laser pulse with an extensive short duration or a very high heat flux. This type of problems has found numerous applications. The method used in this article is applicable to a wide range of problems in thermodynamics. By the obtained results, it is expected that the present model of equations will serve as more realistic and will provide motivation to investigate micropolar thermoelasticity problems.

Conflict of Interest:

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Appendix A:

$b_2 = \frac{\lambda}{\rho c_1^2}$, $b_3 = \frac{2\mu+K}{\rho c_1^2}$, $b_5 = \frac{\mu+K}{\rho c_1^2}$, $b_6 = \frac{\mu}{\rho c_1^2}$, $b_7 = \frac{K}{\rho c_1^2}$, $b_8 = \frac{\omega^{*2}\gamma}{\rho c_1^4}$, $b_9 = \frac{\omega^{*2} b_0}{\rho c_1^4}$, $b_{10} = \frac{\omega^{*2}}{\rho c_1^4}$, $g_{1i} = (m_i^2 - b_2 k^2) + \iota b_3 k m_i \alpha_{3i} - \tau_{11}\alpha_{1i}$,$g_{2i} = -\iota b_3 k m_i + (b_6 m_i^2 + b_5 k^2)\alpha_{3i} - b_7 \alpha_{2i}$, $g_{3i} = -b_8 \alpha_{2i} m_i$

$g_{4i} = -m_i \alpha_{1i}$, $M_1 = \left(\frac{b_1 f_2 + \left(\gamma^{*2} - b_2 k^2\right) f_1 - \tau_{11} f_3 + \iota b_3 k \gamma^* f_4}{f_5}\right)$, $M_2 = \frac{\left(-\iota b_3 k \gamma^* f_1 + \left(b_6 \gamma^{*2} + b_5 k^2\right)\right)}{f_5}$, $M_3 = -\frac{b_8 \gamma^* f_3}{f_5}$, $M_4 = \frac{-\gamma^* f_2}{f_5}$.