

## Ion Acoustic Shock Waves in A Six Component Cometary Plasma

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### Abstract

The effect of pair ions on the formation and propagation characteristics of Ion-Acoustic (IA) shock waves in a six-component cometary plasma composed of two hot and one colder electron component, hot ions, and heavier pair ions is studied. The colder and one hotter component of electrons together with the lighter hydrogen ions are modelled by kappa distributions. The other hotter electron component is described by a q-nonextensive distribution. The KdVB equation is derived for the system and its solution plotted for different kappa values, oxygen ion densities, kinematic viscosities as well as the temperature ratios of ions. In the aforesaid plasma, the shock wave exhibits a transition towards a solitary structure. It is found that the strength of shock profile decreases with an increase in both temperatures of the positively charged oxygen ions and negatively charged oxygen ion densities. However, the strength of the shock wave decreases with a decrease of positively charged oxygen ion densities.

**Keywords:** Ion-Acoustic Shock Wave, KdVB Equation, Kappa Distributions, q-Nonextensive Distribution

### Introduction

Nonlinearity is one of the most beautiful and astonishing manifestations of nature. At large amplitudes, a plasma wave becomes nonlinear. A number of investigations are on-going on nonlinear waves such as shocks, solitons, double-layers, etc. which are observed in space, astrophysical and laboratory plasmas.

The nonlinearity of the ion acoustic (IA) wave has been extensively studied in recent times. The first theoretical investigation on the nonlinear IA wave was by Sagdeev [1]; its experimental observation was confirmed by Ikezi et al. [2].

Every nonlinear phenomena in a plasma is governed by some nonlinear equations. A soliton in which nonlinearity is balanced by dispersion is mainly described by the Korteweg-deVries (KdV) equation. A medium having both dispersion and dissipative effect supports a shock wave instead of a soliton and is described by the Korteweg-deVries-Burgers (KdVB) equation. Dissipative mechanisms such as wave-particle interactions, turbulence, dust charge fluctuations in a dusty plasma, multi-ion streaming, Landau damping, anomalous viscosity, etc. introduce the dissipative Burger term in the nonlinear KdVB equation [3-5]. When wave breaking due to nonlinearity is balanced by the combined effect of dispersion and dissipation, a monotonic or oscillatory dispersive shock wave is generated in a plasma [6]. For a negligible dissipative effect, the solitary wave transforms to a shock wave. The IA shock waves have been extensively studied by many authors recently [7-11].

Most astrophysical plasmas deviate from the well known Maxwellian distribution because of the presence of higher energetic particles. So it is appropriate to describe these plasmas with a non-Maxwellian distribution such as the kappa distribution [12]. Also the extensive formalism fails whenever a physical system includes long-range forces or long-range memory [13]. A nonextensive entropy, which is a generalization of the B-G-S entropy,

was first proposed by Constantino Tsallis [14]. He extended the standard additive nature of entropy to the nonlinear, nonextensive case by introducing a parameter 'q'. This Entropy notion can also be used for goodness of fit for a Maxwellian distribution. See [15-19] for more information on this topic.

Liyan and Jiulin first studied ion acoustic waves in a plasma with the power-law  $q$ -distribution in nonextensive statistics [20]. They proposed that Tsallis statistics is suitable for systems being in the nonequilibrium stationary-state with inhomogeneous temperature and containing a plentiful supply of superthermal particles.

A theoretical investigation of the one dimensional dynamics of nonlinear electrostatic dust ion-acoustic (DIA) waves in an unmagnetized dusty plasma consisting of an ion fluid, non-thermal electrons and fluctuating immobile dust particles has been made by Alinejad [21]. He showed that the special patterns of nonlinear electrostatic waves are significantly modified by the presence of the non-thermal electron component. The transition from DIA solitary to shock waves was also studied, which is related to the contributions of the dispersive and dissipative terms.

Pakzad [22] investigated propagating nonlinear waves in unmagnetized and strongly coupled dusty plasma containing nonthermal ions, Boltzmann distributed electrons and variable dust charge. He found that as long as the dispersive term and the dissipative term as well as the nonlinear term are balanced, a shock wave (both monotonic and oscillatory types) structure forms; otherwise a soliton forms due to the balance between the dispersive and the nonlinear term. In another study, Pakzad again studied the IA shock waves in a plasma consisting of superthermal electrons, Boltzmann distributed positrons and ions by deriving the KdVB equation [23]. It was reported that an increasing positron concentration decreased the amplitude of the waves.

IA solitons have been studied in plasmas where both electrons and ions have been described by  $q$  – nonextensive distributions. Thus, when electrons were described by the Tsallis distribution, it was found that smaller the value of  $q$  ( $q$  is a measure of the deviation of the distribution from the Maxwell-Boltzmann distribution), the greater the width of the soliton [24].

Attention then shifted to dust acoustic (DA) waves in plasmas where both electrons and ions were described by  $q$  - nonextensive distributions [25, 26]. Here too it was found that the soliton width decreased and its amplitude increased when the electron nonextensive parameter  $q \rightarrow 1$ ; the ion nonextensivity made the solitons more spiky. Recently it was shown that in a four component dusty plasma containing non-extensive electrons and two temperature ions both positive and negative polarity solutions existed [27].

A cometary plasma is composed of hydrogen ions, and new born heavier ions and electrons with relative densities depending on their distances from the nucleus. Initially, positively charged oxygen ions were treated as the main heavier ion [28, 29]. However, the discovery of negatively charged oxygen ions [30] enables one to consider the plasma environment around a comet as a pair-ion plasma ( $O^+$ ,  $O^-$ ) with other ions (both lighter and heavier) constituting the other components of the plasma. For instance, the electron distribution in the tail of comet Giacobini-Zinner was observed to have three components: a cold component, a mid component and a hot component - the mid component was interpreted as having a sizeable contribution from the photo-electrons generated by the ionisation of cometary neutrals [31].

We thus model our cometary plasma system, as a six-component plasma consisting of three electron components (two hot and one cold), hydrogen ions, and pair ions [32]. Both kappa and nonextensive electron distributions are used because of their importance in space plasmas and the observation of nonlinear events at Halley's comet [33].

We find that in a six-component cometary plasma with aforesaid components, the nonlinear wave shows a transition from a shock to a soliton. A reduction in the shock wave amplitude is seen with increasing spectral indices and negatively charged oxygen ion densities. The strength of the shock profile also decreases with increasing temperatures of the positively charged oxygen ions and viscosity of negatively charged oxygen ions.

### Basic Equations

We consider the existence of Ion-Acoustic shock waves in a six component plasma consisting of negatively and positively charged oxygen ions (represented, respectively, by subscripts '1' and '2'), kappa described hydrogen ions, hot electrons of solar origin and colder electrons of cometary origin. The second hot electron of cometary origin is described by a q-nonextensive distribution. At equilibrium, charge neutrality requires that

$$n_{ce0} + n_{he0} + n_{se0} + Z_1 n_{10} = n_{H0} + Z_2 n_{20} \tag{1}$$

In the above equation  $n_{ce0}$ ,  $n_{he0}$  represent the equilibrium densities of colder and hotter cometary electrons respectively whereas  $n_{se0}$  represents the equilibrium density of hotter solar electrons. Also  $n_{10}$ ,  $n_{20}$ ,  $n_{H0}$  are respectively, the equilibrium densities of negatively charged oxygen (O<sup>-</sup>) ions, positively charged oxygen (O<sup>+</sup>) ions and hydrogen ions.  $Z_1$  and  $Z_2$  denote the charge numbers of O<sup>-</sup> and O<sup>+</sup> ions respectively.

The dynamics of the heavier ions can be described by the following hydrodynamic equations:

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j v_j)}{\partial x} = 0 \tag{2}$$

$$\left( \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x} \right) v_j = m \frac{Z_j e}{m_j} \frac{\partial \phi}{\partial x} - \frac{1}{m_j n_j} \frac{\partial P_j}{\partial x} + \mu_j \frac{\partial^2 v_j}{\partial x^2} \tag{3}$$

where '-' sign refers to positively charged oxygen ions (and vice versa) and  $v_j$  and  $m_j$ , respectively, denote the fluid velocity and mass of the j-species of ions (j=O<sup>-</sup>, O<sup>+</sup>).

In (3) the adiabatic equation of state for ions, is  $\frac{P_j}{P_{j0}} = \left( \frac{n_j}{n_{j0}} \right)^\gamma = N_j^\gamma$  where

$P_{j0} = n_{j0} k_B T_j$ ,  $\gamma = (N+2)/N$  for an N dimensional system and  $\mu_j$  is the ion kinematic viscosity. Here we are considering a one dimensional system and hence  $\gamma = 3$ .

The Poisson's equation is given by

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e (n_H + Z_2 n_2 - Z_1 n_1 - n_{ce} - n_{he} - n_{se}) \tag{4}$$

We normalize (2) to (4) using the parameters of O<sup>-</sup> ions according to,  $\phi = \frac{e\phi}{k_B T_1}$ ,  $V_j = \frac{v_j}{c_s}$ ,  $\tau = \frac{t}{\omega_{p1}^{-1}}$  where

$c_s = \left( \frac{Z_1 k_B T_1}{m_1} \right)^{1/2}$  and  $\omega_{p1} = \left( \frac{4\pi Z_1^2 e^2 n_{10}}{m_1} \right)^{1/2}$ . The variable x is normalized using  $\lambda_{D1} = \left( \frac{Z_1 k_B T_1}{4\pi Z_1^2 e^2 n_{10}} \right)^{1/2}$  while

$$N_j = \frac{n_j}{n_{j0}}$$

Thus, equations (2) to (4) can be rewritten as

$$\frac{\partial N_j}{\partial \tau} + \frac{\partial(N_j V_j)}{\partial x} = 0 \quad j = 1,2 \tag{5}$$

$$\frac{\partial V_1}{\partial \tau} + V_1 \frac{\partial V_1}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{3N_1}{Z_1} \frac{\partial N_1}{\partial x} + \rho_1 \frac{\partial^2 V_1}{\partial x^2} \tag{6}$$

$$\frac{\partial V_2}{\partial \tau} + V_2 \frac{\partial V_2}{\partial x} = \frac{-Z_2 m}{Z_1} \frac{\partial \phi}{\partial x} - \frac{3m\beta N_2}{Z_1} \frac{\partial N_2}{\partial x} + \rho_2 \frac{\partial^2 V_2}{\partial x^2} \tag{7}$$

where

$$m = \frac{m_1}{m_2}, \quad \beta = \frac{T_2}{T_1}, \quad \rho_1 = \frac{\mu_1}{\omega_{p1} \lambda_D^2} \quad \text{and} \quad \rho_2 = \frac{\mu_2}{\omega_{p1} \lambda_D^2}$$

$\rho_1$  and  $\rho_2$  now represent the normalized kinematic viscosities of the pair ions.

The normalised kappa distribution of solar electrons, colder cometary electrons and hydrogen ions are given by,

$$n_k = n_{se0} \left[ 1 - \frac{\phi}{\sigma_k (\kappa_k - 3/2)} \right]^{-\kappa_k + 1/2} \quad (\text{k = 'se' or 'ce'}) \tag{8}$$

$$n_H = n_{H0} \left[ 1 + \frac{\phi}{\sigma_H (\kappa_H - 3/2)} \right]^{-\kappa_H + 1/2} \tag{9}$$

One of the hot electron components is modelled using q non-extensive distribution and is given by

$$n_{he} = \left( 1 + \sigma_{he} (q-1) \phi \right)^{\frac{3q-1}{2(q-1)}} \tag{10}$$

The normalized Poisson's equation after substitution of (8) - (10) to (4) is,

$$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2} = & N_1 - N_2 (1 + \mu_{ce} + \mu_{se} + \mu_{he} - \mu_H) + \mu_1 (1 + (q-1) \sigma_{he} \phi)^{\frac{3q-1}{2(q-1)}} + \mu_{ce} \left( 1 - \frac{\phi}{\sigma_{ce} (\kappa_{ce} - 3/2)} \right)^{-(\kappa_{ce} - 1/2)} \\ & + \mu_{se} \left( 1 - \frac{\phi}{\sigma_{se} (\kappa_{se} - 3/2)} \right)^{-(\kappa_{se} - 1/2)} - \mu_H \left( 1 + \frac{\phi}{\sigma_H (\kappa_H - 3/2)} \right)^{-(\kappa_H - 1/2)} \end{aligned} \tag{11}$$

where  $\mu_1 = \frac{1}{Z_1 n_{10}}$ ,  $\mu_{ce} = \frac{n_{ce0}}{Z_1 n_{10}}$ ,  $\mu_{se} = \frac{n_{se0}}{Z_1 n_{10}}$ ,  $\mu_{he} = \frac{n_{he0}}{Z_1 n_{10}}$ ,  $\mu_H = \frac{n_{H0}}{Z_1 n_{10}}$ ,  $\sigma_{ce} = \frac{T_{ce}}{T_1}$ ,  $\sigma_{he} = \frac{T_1}{T_{he}}$ ,  $\sigma_{se} = \frac{T_{se}}{T_1}$  and  $\sigma_H = \frac{T_H}{T_1}$

**Derivation of KdVB Equation**

We use the reductive perturbation method to derive the KdVB equation from (5) to (11) by introducing the transformations [34]

$$\xi = \varepsilon^{1/2} (x - \lambda t), \tau = \varepsilon^{3/2} t, \rho_j = \varepsilon^{1/2} \rho_{j0}$$

where  $\varepsilon$  is a smallness parameter and  $\lambda$  is the wave phase speed.

To apply the reductive perturbation technique the various parameters are expanded as

$$N_{1,2} = 1 + \varepsilon N_{1,2}^{(1)} + \varepsilon^2 N_{1,2}^{(2)} + \dots \tag{12}$$

$$V_{(1,2)} = \varepsilon V_{(1,2)}^{(1)} + \varepsilon^2 V_{(1,2)}^{(2)} + \dots \tag{13}$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \dots \tag{14}$$

We substitute (12) to (14) in (5) to (11) and equate the coefficients of different powers of  $\varepsilon$ . From the coefficients of order  $\varepsilon^{3/2}$  we get the first order terms as,

$$N_1^1 = \frac{\phi^1}{\left(\frac{3}{Z_1} - \lambda^2\right)} \tag{15}$$

and

$$N_2^1 = \frac{(Z_2 / Z_1) m \phi^1}{\left(\lambda^2 - \frac{3m\beta}{Z_1}\right)} \tag{16}$$

Expressions for  $V_1^1$  and  $V_2^1$  can be obtained by multiplying (15) and (16) by  $\lambda$ .

Also, the linear dispersion relation is

$$\lambda^2 = \frac{S \pm \sqrt{S^2 - 12 m Z_1^2 T [3m\beta T + mZ_2 (1 + \mu_{ce} + \mu_{he} + \mu_{se} - \mu_H) + m\beta Z_1]}}{2TZ_1^2}$$

where  $S = Z_1^2 + mZ_1Z_2(1 + \mu_{ce} + \mu_{se} - \mu_H) + 3Z_1T(1 + m\beta)$  (17)

and

$$T = \frac{\mu_{ce}(\kappa_{ce} - 1/2)}{(\kappa_{ce} - 3/2)\sigma_{ce}} + \frac{\mu_{se}(\kappa_{se} - 1/2)}{(\kappa_{se} - 3/2)\sigma_{se}} + \frac{\mu_H(\kappa_H - 1/2)}{(\kappa_H - 3/2)\sigma_H} + \frac{\mu_1(3q-1)\sigma_{he}}{2}$$

Equating the coefficients of  $\epsilon^{5/2}$  in (5), we get

$$\frac{\partial N_j^1}{\partial \tau} - \lambda \frac{\partial N_j^2}{\partial \xi} + \frac{\partial V_j^2}{\partial \xi} + \frac{\partial(N_j^1 V_j^1)}{\partial \xi} = 0 \quad j = 1, 2 \quad (18)$$

And the coefficient of order  $\epsilon^{5/2}$  in (6) and (7) results in,

$$\frac{\partial V_1^1}{\partial \tau} - \lambda \frac{\partial V_2^2}{\partial \xi} + V_1^1 \frac{\partial V_1^1}{\partial \xi} = \frac{\partial \phi^2}{\partial \xi} - \frac{3N_1^1}{Z_1} \frac{\partial N_1^1}{\partial \xi} - \frac{3}{Z_1} \frac{\partial N_2^2}{\partial \xi} + \rho_{10} \frac{\partial^2 V_1^1}{\partial \xi^2} \quad (19)$$

$$\frac{\partial V_2^1}{\partial \tau} - \lambda \frac{\partial V_2^2}{\partial \xi} + V_2^1 \frac{\partial V_2^1}{\partial \xi} = -\frac{Z_2 m}{Z_1} \frac{\partial \phi^2}{\partial \xi} - \frac{3m\beta N_2^1}{Z_1} \frac{\partial N_2^1}{\partial \xi} - \frac{3m\beta}{Z_1} \frac{\partial N_2^2}{\partial \xi} + \rho_{20} \frac{\partial^2 V_2^1}{\partial \xi^2} \quad (20)$$

Finally, equating the coefficients of terms of order  $\epsilon^2$  from Poisson's equation (11) gives,

$$\begin{aligned} \frac{\partial^2 \phi^1}{\partial \xi^2} = & N_1^2 - N_2^2(1 + \mu_{ce} + \mu_{he} + \mu_{se} - \mu_H) + T\phi^2 + \frac{\mu_{ce}}{2} \frac{(\kappa_{ce}^2 - 1/4)}{(\kappa_{ce} - 3/2)^2 \sigma_{ce}^2} (\phi^1)^2 + \frac{\mu_{se}}{2} \frac{(\kappa_{se}^2 - 1/4)}{(\kappa_{se} - 3/2)^2 \sigma_{se}^2} (\phi^1)^2 \\ & - \frac{\mu_H}{2} \frac{(\kappa_H^2 - 1/4)}{(\kappa_H - 3/2)^2 \sigma_H^2} (\phi^1)^2 + \frac{\mu_1}{4} (3q-1)(q+1)\sigma_{he}^2 (\phi^1)^2 \end{aligned} \quad (21)$$

Substituting the values from (15) and (16) into (18) to (21) and eliminating the second order terms, we obtain the KdVB equation as

$$A \frac{\partial \phi^1}{\partial \tau} + \phi^1 \frac{\partial \phi^1}{\partial \xi} + B \frac{\partial^3 \phi^1}{\partial \xi^3} - C \frac{\partial^2 \phi^1}{\partial \xi^2} = 0 \quad (22)$$

In (22) the coefficients A, B, and C are given by

$$A = \frac{-2Z_1^2 \lambda (Z_1 \lambda^2 - 3)(Z_1 \lambda^2 - 3m\beta) \left[ (Z_1 \lambda^2 - 3m\beta)^2 + m \left( \frac{Z_2}{Z_1} \right) (1 + \mu_{ce} + \mu_{he} + \mu_{se} - \mu_H) (Z_1 \lambda^2 - 3)^2 \right]}{D}$$

$$B = \frac{-(Z_1\lambda^2 - 3)^3 (Z_1\lambda^2 - 3m\beta)^3}{D}$$

$$C = \frac{-Z_1^2\lambda(Z_1\lambda^2 - 3)(Z_1\lambda^2 - 3m\beta) \left[ \rho_{10}(Z_1\lambda^2 - 3m\beta)^2 - m\rho_{20} \left( \frac{Z_2}{Z_1} \right) (1 + \mu_{ce} + \mu_{he} + \mu_{se} - \mu_H)(Z_1\lambda^2 - 3)^2 \right]}{D}$$

where

$$L = \frac{\mu_{ce}(\kappa_{ce}^2 - 1/4)}{(\kappa_{ce} - 3/2)^2 \sigma_{ce}^2} + \frac{\mu_{se}(\kappa_{se}^2 - 1/4)}{(\kappa_{se} - 3/2)^2 \sigma_{se}^2} + \frac{\mu_1}{2}(3q-1)(q+1)\sigma_{he}^2 - \frac{\mu_H(\kappa_H^2 - 1/4)}{(\kappa_H - 3/2)^2 \sigma_H^2}$$

and

$$D = L(Z_1\lambda^2 - 3)^3 (Z_1\lambda^2 - 3m\beta)^3 + 3Z_1^2(1 + Z_1\lambda^2)(Z_1\lambda^2 - 3m\beta)^3 - 3m^2Z_2^2(1 + \mu_{ce} + \mu_{he} + \mu_{se} - \mu_H)(Z_1\lambda^2 - 3)(Z_1\lambda^2 + m\beta) \tag{23}$$

The above results in (22) and (23) are comparable with the results of Pakzad [23] for a plasma consisting of superthermal electrons, positrons and ions. Equation (22) is the well known KdVB equation describing the nonlinear propagation of the ion acoustic shock waves in a plasma with superthermal electrons. In this equation B is the dispersive term and the Burger term C arises due to the effect of heavy ions kinematic viscosity. In the absence of the viscosity term, (22) reduces to the usual KdV equation for the propagation of ion acoustic solitary waves where nonlinearity is balanced by dispersive effects. On the other hand, if the coupling becomes very strong the shock waves will appear. The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B and C, respectively.

### Solution of KdVB Equation

In order to find the solution of (22) we use the transformed coordinate  $\chi = (\xi - V\tau)$  of the comoving frame with speed V and use the boundary conditions:  $\phi^1 \rightarrow 0$  and  $\frac{\partial \phi^1}{\partial \chi}, \frac{\partial^2 \phi^1}{\partial \chi^2}, \frac{\partial^3 \phi^1}{\partial \chi^3} \rightarrow 0$  as  $\chi \rightarrow \infty$  for a localized solution [35].

When the partial differential equation of a system is formed by the combined effect of dispersion and dissipation, a convenient method to solve it is "the tanh method" [36, 37]. Using the above transformation (22) can be written as,

$$-AV \frac{\partial \phi^1}{\partial \chi} + \phi^1 \frac{\partial \phi^1}{\partial \chi} + B \frac{\partial^3 \phi^1}{\partial \chi^3} - C \frac{\partial^2 \phi^1}{\partial \chi^2} = 0 \tag{24}$$

Again using the transformation  $\alpha = \tanh \chi$  and assuming a series solution of the form  $\phi^1(\alpha) = \sum_{i=0}^n a_i \alpha^i$ , we arrive at the solution of (24), as

$$\phi^1 = AV + 8Bk^2 + \frac{C^2}{25B} - 12k^2 \tanh^2[k(\xi - V\tau)] - \frac{12}{5}kC[1 - \tanh[k(\xi - V\tau)]] \quad (25)$$

The speed of comoving frame is related to the coefficients A, B and C as  $V = \frac{(100B^2k^2 - C^2 + 60kBC)}{25AB}$  and

$k = \frac{\pm C}{10B}$  which can be obtained using the boundary conditions.

### Results

Solution (25) is applicable to any astrophysical plasma. But, in this paper, we concentrate on parameters relevant to comet Halley: the observed value of the density of hydrogen ions was  $n_H = 4.95 \text{ cm}^{-3}$ ; their temperature was  $T_H = 8 \times 10^4 \text{ K}$ . The temperature of the solar (or hot) electrons was  $T_{se} = 2 \times 10^5 \text{ K}$  [38]. The temperature of the second component of the photo-electron was set at  $T_{ce} = 2 \times 10^4 \text{ K}$ . Negatively charged oxygen ions with an energy  $\sim 1 \text{ eV}$  and densities  $\leq 1 \text{ cm}^{-3}$  was identified by Chaizy et al. [30]. We thus set the densities of positively charged oxygen ions at  $n_{20} = 0.5 \text{ cm}^{-3}$  and that of negatively charged oxygen ions at  $n_{10} = 0.05 \text{ cm}^{-3}$  [38, 3].

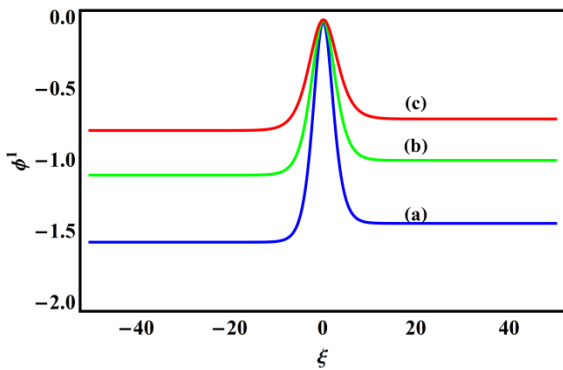


Fig. 1

Shock profile variation as a function of temperature of  $O^+$  ions

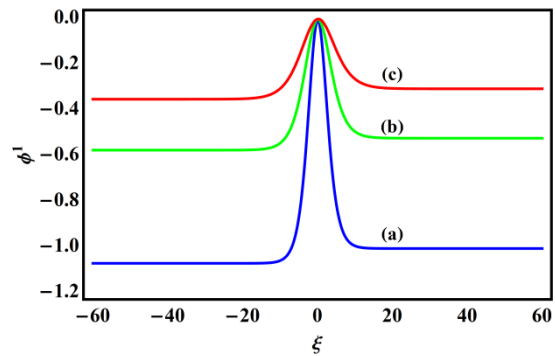


Fig. 2

Shock profile variation as a function of density of  $O^-$  ions

Figure 1 is a plot of the solution (25) of KdVB equation (24), and shows the variation of the temperature of positively charged oxygen ions; the parameters for the figure are  $n_{10} = 0.05 \text{ cm}^{-3}$ ,  $n_{20} = 0.5 \text{ cm}^{-3}$ ,  $n_H = 4.95 \text{ cm}^{-3}$ ,  $T_{ce} = 2 \times 10^4 \text{ K}$ ,  $T_{se} = 2 \times 10^5 \text{ K}$ ,  $T_{he} = 9 \times 10^4 \text{ K}$ ,  $T_1 = 1.16 \times 10^4 \text{ K}$ ,  $Z_1 = 1$ ,  $Z_2 = 2$ ,  $\kappa_{se} = \kappa_{ce} = \kappa_H = 3$ ,  $\rho_{10} = 0.1$ ,  $\rho_{20} = 0.75$ ,  $\tau = 0.2$ ,  $q = 0.2$ . Curve (a) is for  $T_2 = 1.9 \times 1.16 \times 10^4 \text{ K}$ , curve (b) is for  $T_2 = 2 \times 1.16 \times 10^4 \text{ K}$  and curve (c) is for  $T_2 = 2.1 \times 1.16 \times 10^4 \text{ K}$ . It is clear from the figure that the strength of shock profile decreases with an increase in the temperature of the positively charged  $O^+$  ions and as the temperature increases the shock wave nature gradually transforms to soliton.



Figure 2 depicts the variation of the potential  $\phi^1$  versus  $\xi$  as a function of density of negatively charged oxygen ion; for this figure we fix  $T_2 = 2 \times 1.16 \times 10^4 K$ . All other parameters are the same as in figure 1. Curve (a) is for  $n_{10} = 0.03 \text{ cm}^{-3}$ , curve (b) is for  $n_{10} = 0.04 \text{ cm}^{-3}$ , curve (c) is  $n_{10} = 0.05 \text{ cm}^{-3}$ . We find that the strength of the shock wave decreases with an increase of the negatively charged oxygen ion density. As the density of negatively charged oxygen ions increase, the shock wave completely transforms to soliton.

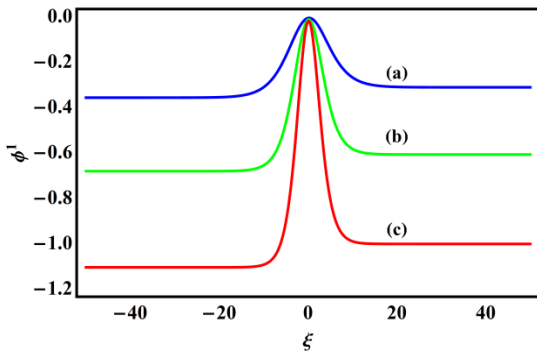


Fig. 3

Shock profile variation as a function of density of  $O^+$  ions

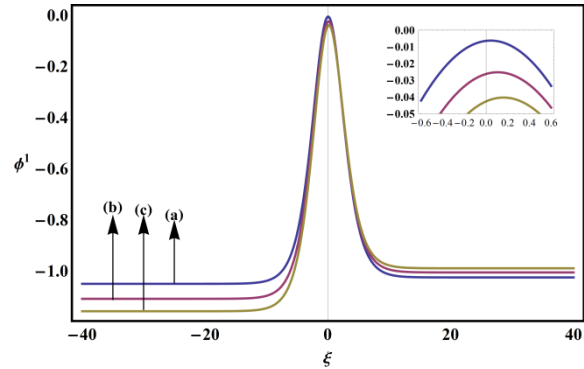


Fig. 4

Shock profile variation as a function of kappa indices

Figure 3 again is a plot of the solution (25), and shows the variation of the potential  $\phi^1$  versus  $\xi$  as a function of density of positively charged oxygen ions; the parameters for the figure are again the same as in figure 1. Curve (a) is for  $n_{20} = 0.3 \text{ cm}^{-3}$ , curve (b) is for  $n_{20} = 0.4 \text{ cm}^{-3}$ , curve (c) is for  $n_{20} = 0.5 \text{ cm}^{-3}$ . We find that the strength of the shock wave decreases with a decrease of the positively charged oxygen ion density. At lower  $O^+$  ion density, shock wave transforms to soliton. In terms of electron densities, Figures 2 and 3 allows one to conclude that as the electron densities decrease the shock wave transforms to a solitary wave.

Figure 4 represents the shock profile as a function of kappa indices; the parameters for the figure are the same as in figure 1. Curve (a) is for the spectral index  $\kappa_{se} = \kappa_{ce} = \kappa_H = 2$ , curve (b) is for  $\kappa_{se} = \kappa_{ce} = \kappa_H = 3$  and curve (c) is for  $\kappa_{se} = \kappa_{ce} = \kappa_H = 4$ . It is clear from the figure that the amplitude of the shock wave increases with the decrease of kappa indices. i.e., the superthermality of se, ce, and  $H^+$  enhances the strength of the shock wave.

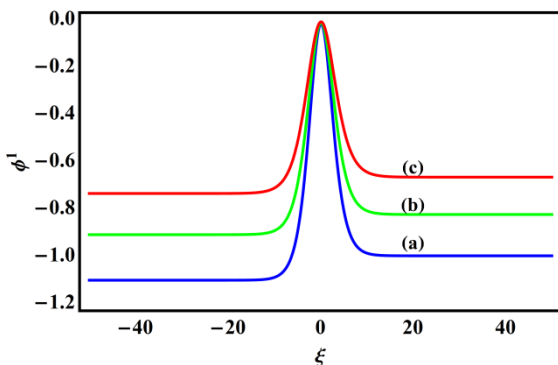


Fig. 5

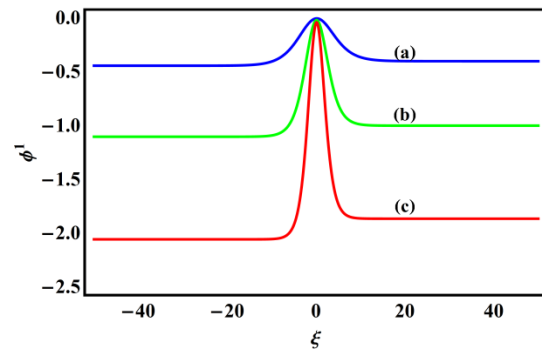


Fig. 6

Shock profile variation as a function of kinematic viscosity of O<sup>-</sup> ions

Shock profile variation as a function of kinematic viscosity of O<sup>+</sup> ions

Figure 5 is again a plot of the solution (25), and shows the variation of the potential  $\phi^1$  versus  $\xi$  as a function of kinematic viscosity of negatively charged oxygen ions; the parameters for the figure are the same as in figure 1. Curve (a) is for  $\rho_{10} = 0.1$ , curve (b) is for  $\rho_{10} = 0.2$  and curve (c) for  $\rho_{10} = 0.3$ . We find that the strength of the shock wave decreases with an increase of the kinematic viscosity of negatively charged oxygen ion and we can see that there is a gradual change in nature from a shock to a soliton.

Figure 6 indicates the variation of the potential  $\phi^1$  versus  $\xi$  as a function of kinematic viscosity of positively charged oxygen ion. Curve (a) is for  $\rho_{20} = 0.5$ , curve (b) is for  $\rho_{20} = 0.75$  and curve (c) for  $\rho_{20} = 1$ . We find that the strength of the shock wave increases with an increase of the kinematic viscosity of positively charged oxygen ions.

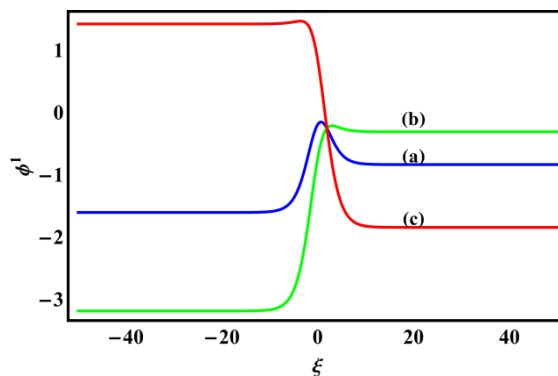


Fig. 7

Shock profile variation as a function of temperature of cold electrons

Figure 7 depicts the variation of the potential  $\phi^1$  versus  $\xi$  as a function of temperature of cold electrons; the parameters chosen are the same as in figure 1. Curve (a) is for  $T_{ce} = 3.5 \times 10^4$  K, curve (b) is for  $T_{ce} = 4 \times 10^4$  K and curve (c) for  $T_{ce} = 4.5 \times 10^4$  K. As the temperature of the cold electron component increases, there is a rapid transition from solitary structure to shock.

### Conclusions

We have investigated the shock wave profiles in a six component plasma consisting of solar and cometary components by deriving the KdVB equation. The solar component consists of lighter hydrogen ions and a hot (superthermal) electron component. The cometary contribution consists of a pair of heavier ion components, colder superthermal electrons, and hot nonextensive electrons. The influence of spectral indices kappa, temperature of positively charged oxygen ions and density of oxygen ions (O<sup>+</sup>, O<sup>-</sup>) on the shock wave profile has been studied. We find that in a six component cometary plasma with aforesaid components, the nonlinear wave shows a transition from a shock to a soliton. A reduction in the shock wave amplitude is seen with increasing spectral indices and negatively charged oxygen ion densities. The strength of the shock profile also decreases with increasing temperatures of the positively charged oxygen ions and viscosity of negatively charged oxygen ions. However, it increases with an increase of the kinematic viscosity of positively charged oxygen ions. Heavy ions were surmised to effect nonlinear waves and nonlinear solitary waves have been observed in the environment of comet Halley. Our results show that the amplitude of the solitary waves seems

to be well correlated to the presence of water molecules in a cometary plasma and the associated photo-ionisation processes.

Interestingly the two components of electrons observed at comet 67P/ Churyumov-Gerasimenko were recently modelled by kappa distributions [39]. Also ions such as  $H^+$ ,  $H^-$ ,  $O^-$ , and  $O^+$  were identified in the coma of 67P/ Churyumov-Gerasimenko. Hence our plasma model is applicable to the environment around the coma of the comet 67P/ Churyumov-Gerasimenko.

The six component model used very well describes the plasma environment around a comet and such analytic studies serve to emphasise the role of each component in the generation of nonlinear events. However, future explorations should endeavour to simultaneously measure both particle distributions as well as nonlinear events so that experimental observations and analytic and modelling studies complement one another.

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