On A Chain Involving the Multivariable I-Transform I

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Abstract

In the present paper we first establish an interesting new chain interconnecting a number of multivariable I-transform of Prathima et al. [6] by the method of mathematical induction. Full care has been taken of all the convergence and existence conditions for the validity of the chain. The chain established herein has been put in a very compact form and it exhibits interesting relationship existing between images and originals of a series of related functions in several multidimensional I-function. The importance of our findings lies in the fact that it involves the multivariable I-function which is sufficiently general in nature and so a large number of chains involving other simpler and useful integral transforms of one and more variables follow as special ceses of our chain merely by specializing the parameters. In the end, we shall see several corollaries.

Keywords: Chain for Integral Transform, Multivariable I-Function, Multidimensional I-Transform.

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1. Introduction.

Inspired by the usefulness of integrals transforms in one and two variables in obtained solution of boudary value problems occurring in various field of Physics and Engineering, we aim here to study certain properties of a multidimensional integral transform whose kernel involves the I-function of severnal variables. This integral transform provides interesting unifications and extensions of the various classes of known E,G, H-functions and I-functions of one and two variables of the product of several such functions as studied by Srivastava [9], Brychkov et al. [1] have written a standard test on multidimensional integral transforms giving a good introduction and applications of such transforms(see [3]. p.473). In the present study we establish an interesting new chain interconnecting number of multivariable I-function of Prathima et al. [6] and discuss its several special cases. The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,\mathbf{n}:m_1,n_1;\cdots;m_r,n_r}_{\mathbf{p},\mathbf{q}:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{array}\middle|\begin{array}{l} (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p} : \\ (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q} : \end{array}\right.$$

$$\left.\begin{array}{l} (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

where $\phi(s_1,\cdots,s_r)$, $\theta_i(s_i)$, $i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.2}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.3}$$

For more details, see Prathima et al. [6]. The I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{p}A_j\alpha_j^{(i)} - \sum_{j=1}^{q}B_j\beta_j^{(i)} + \sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)} \leqslant 0, i=1,\cdots,r \tag{1.4}$$

The integral (1.1) converges absolutely if

$|arg(z_k)| < \frac{1}{2}\Delta_k\pi, k=1,\cdots,r$ where

$$\Delta_k = -\sum_{j=n+1}^{p}A_j\alpha_j^{(k)} - \sum_{j=1}^{q}B_j\beta_j^{(k)} + \sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)} - \sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)} + \sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)} > 0 \tag{1.5}$$

Following the lines of Braaksma ([3] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r) = 0(\ |z_1|^{\alpha_1},\cdots,\ |z_r|^{\alpha_r}\ )\ ,\ max(\ |z_1|,\cdots,\ |z_r|\ ) \to 0$$

$$I(z_1,\cdots,z_r) = 0(\ |z_1|^{\beta_1},\cdots,\ |z_r|^{\beta_r}\ )\ ,\ min(\ |z_1|,\cdots,\ |z_r|\ ) \to \infty$$

where $i=1,\cdots,r$:

$$\alpha_i = \min_{1\leqslant j\leqslant m_i} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i = \max_{1\leqslant j\leqslant n_i} Re\left[C_j^{(i)}\left(\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)\right]$$

The multivariable I-function was introduced ans studies by Prathima et al. [6]. In this paper we shall study a special case of this function defined and represented in the following manner

$$\bar{I}^{(k)}[z_1^{(k)},\cdots,z_r^{(r)}] = \bar{I}_{p^{(k)},q^{(k)};p_1^{(k)},q_1^{(k)}\cdots;p_r^{(k)},q_r^{(k)}}^{0,0:m_1^{(k)},n_1^{(k)};\cdots;m_r^{(k)},n_r^{(k)}}$$

$$\left(\begin{array}{c|c} \mathrm{z}_1^{(k)} & (\mathrm{a}_j^{(k)};\alpha_{1j}^{(k)},\cdots,\alpha_{rj}^{(k)},A_j^{(k)})_{1,p^{(k)}}:(c_{1j}^{(k)},\gamma_{1j}^{(k)};C_{1j}^{(k)})_{1,p_1^{(k)}},\cdots,(c_{rj}^{(k)},\gamma_{rj}^{(k)};C_{rj}^{(k)})_{1,p_r^{(k)}}. \\ \cdot & \cdot \\ \cdot & \\ \mathrm{z}_r^{(k)} & (\mathrm{b}_j^{(k)};\beta_{1j}^{(k)},\cdots,\beta_{rj}^{(k)},B_j^{(k)})_{1,q^{(k)}}:(d_{1j}^{(k)},\delta_{1j}^{(k)};D_{1j}^{(k)})_{1,q_1^{(k)}},\cdots,(d_{rj}^{(k)},\delta_{rj}^{(k)};D_{rj}^{(k)})_{1,q_r^{(k)}} \end{array}\right)$$

$$=$$

$$\frac{1}{(2\pi\omega)^r}\int_{L_1^{(k)}}\cdots\int_{L_r^{(k)}}\theta_1^{(k)}(\xi_r^{(k)})\cdots\theta_r^{(k)}(\xi_r^{(k)})\psi^{(k)}(\xi_1^{(k)},\cdots,\xi_r^{(k)})(z_1^{(k)})^{\xi_1^{(k)}}\cdots(z_r^{(k)})^{\xi_r^{(k)}}d\xi_1^{(k)}\cdots d\xi_r^{(k)} \tag{1.6}$$

where

$$\psi^{(k)}(\xi_1^{(k)},\cdots,\xi_r^{(k)})=\left[\prod_{j=1}^{q^{(k)}}\Gamma^{B_j^{(k)}}\left(1-b_j^{(k)}+\sum_{i=1}^{r}\beta_{ij}^{k)}\xi_i^{(k)}\right)\prod_{j=1}^{p^{(k)}}\Gamma^{A_j^{(k)}}\left((a_j^{(k)}-\sum_{i=1}^{r}\alpha_{ij}^{(k)}\xi_i^{(k)}\right)\right]^{-1} \tag{1.7}$$

and

Throughout the paper we have used the following notations.

(1.9)

where

$$C_k=(c'_{1j},\gamma'_{1j};C'_{1j})_{1,n'_1},\quad\left[\left(c_{1j}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_1^{(i)}+l-1\right]\gamma_{1j}^{(l)},\gamma_{1j}^{(l)};C_{1i}^{(l)}\right)_{1,n_1^{(l)}}\right]_{l=2}^{k-1},$$

$$\left(c_{1j}^{(k)}+\left[\sum_{i=1}^{k-1}\rho_1^{(i)}+k-1\right]\gamma_{1j}^{(k)},\gamma_{1j}^{(k)};C_{1j}^{(k)}\right)_{1,p_1^{(k)}},$$

$$(c'_{1j},\gamma'_{1j};C'_{1j})_{n'_1+1,p'_1},\quad\left[\left(c_{1i}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_1^{(i)}+l-1\right]\gamma_{1j}^{(l)},\gamma_{1j}^{(l)};C_{1i}^{(l)}\right)_{1+n_1^{(l)},p_1^{(l)}}\right]_{l=2}^{k-1},\cdots,$$

$$(c'_{rj},\gamma'_{rj};C'_{rj})_{1,n'_r},\quad\left[\left(c_{ri}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_r^{(i)}+l-1\right]\gamma_{rj}^{(l)},\gamma_{rj}^{(l)};C_{ri}^{(l)}\right)_{1,n_r^{(l)}}\right]_{l=2}^{k-1},$$

$$\left(c_{rj}^{(k)}+\left[\sum_{i=1}^{k-1}\rho_r^{(k)}+k-1\right]\gamma_{rj}^{(k)},\gamma_{rj}^{(k)};C_{rj}^{(k)}\right)_{1,p_r^{(k)}},$$

$$\left(c_{1j}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_u^{(i)}+l-1\right]\gamma_{1j}^{(l)},\gamma_{1j}^{(l)};C_{1j}^{(l)}\right]_{1,p_1^{(k)}} , (c'_{rj},\gamma'_{rj};C'_{rj})_{n'_r+1,p'_r}$$

$$D_k=(d'_{1j},\delta'_{1j};D'_{1j})_{1,m'_1},\ \left[\left(d_{1j}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_1^{(i)}+l-1\right]\delta_{1j}^{(l)},\delta_{1j}^{(l)};D_{1i}^{(l)}\right)_{1,m_1^{(l)}}\right]_{l=2}^{k-1},$$

$$\left(d_{1j}^{(k)}+\left[\sum_{i=1}^{k-1}\rho_1^{(k)}+k-1\right]\delta_{1j}^{(k)},\delta_{1j}^{(k)};D_{1j}^{(k)}\right)_{1,q_1^{(k)}},$$

$$(d'_{1j},\delta_{1j};D'_{1j})_{m'_1+1,q'_1},\ \left[\left(d_{1i}^{(l)}+\left[\sum_{i=1}^{l-1}\rho_1^{(i)}+l-1\right]\delta_{1j}^{(l)},\delta_{1j}^{(l)};D_{1i}^{(l)}\right)_{1,m_1^{(l)}}\right]_{l=2}^{k-1},\cdots,$$

,

$$\left(d_{rj}^{(k)}+\left[\sum_{i=1}^{k-1}\rho_r^{(k)}+k-1\right]\delta_{rj}^{(k)},\delta_{rj}^{(k)};D_{rj}^{(k)}\right)_{1,q_r^{(k)}},(d'_{1j},\delta'_{1j};D'_{1j})_{,m'_r+1,q'_r},$$

The multidimensional I-transform occurring in the paper is defined by

$$\eta^{(k)}(s_1,\cdots,s_r)=\bar{I}^{(k)}[f(t_1,\cdots,t_r);s_1,\cdots,s_r]=\int_0^\infty\cdots\int_0^\infty\bar{I}^{(k)}[s_1t_1,\cdots,s_rt_r]f(t_1,\cdots,t_r)\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.14}$$

provided that the multiple integral (1.8) is absolutely convergent. The following result which is a modified form of a known result [4.p.23, eq.(2.1)] will be required in the sequel.

$$\int_0^\infty\cdots\int_0^\infty(\prod_{u=1}^{r}(t_u^{(k)})^{\left(\sum_{i=1}^{k-1}\rho_u^{(i)}+k-2\right)}\bar{I}_{k-1}[t'_1(t_1^{(k)})^{-1},\cdots,t'_r(t_r^{(k)})^{-1}]\bar{I}^{(k)}[s_1t_1^{(k)},\cdots,s_rt_r^{(k)}]dt_1^{(k)}\cdots dt_r^{(k)}$$

The above multiple integrals converge under the following conditions if

$$U_i^{(k)}>0,V_i^{(k)}>0,|arg(s_i)|<\frac{1}{2}U_i^{(k)}\pi \tag{1.16}$$

$$\sum_{l=1}^{k-1}\max_{1\leq j\leq n_i^{(l)}}Re\left[C_{ij}^{(l)}\frac{c_{ij}^{(l)}-1}{\gamma_{ij}^{(l)}}+\sum_{u=1}^{l-1}\rho_i^{(u)}+l-1\right]-\min_{1\leq j\leq m_i^{(k)}}D_{ij}^{(k)}Re\frac{d_{ij}^{(k)}}{\delta_{ij}^{(k)}}<\ Re\left[\sum_{u=1}^{l-1}\rho_i^{(u)}+l-1\right]<$$

$$\sum_{l=1}^{k-1} \max_{1\leq j\leq n_i^{(l)}} Re\left[D_{ij}^{(l)}\frac{d_{ij}^{(l)}}{\delta_{ij}^{(l)}}+\sum_{u=1}^{l-1}\rho_i^{(u)}+l-1\right]-\min_{1\leq j\leq m_i^{(k)}} C_{ij}^{(k)} Re\frac{c_{ij}^{(k)}-1}{\gamma_{ij}^{(k)}} \tag{1.17}$$

where

$$U_i^{(k)}=\sum_{j=1}^{p^{(k)}}A_{ij}^{(k)}a_{ij}^{(k)} - \sum_{j=1}^{q_i^{(k)}}B_{ij}^{(k)}\beta_{ij}^{(k)}+\sum_{j=1}^{m^{(k)}}D_{ij}^{(k)}\delta_{ij}^{(k)}-\sum_{j=m_i^{[k)}+1}^{q_i^{(k)}}D_{ij}^{(k)}\delta_{ij}^{(k)}-\sum_{j=m_i^{(k}+1}^{q_i^{(k)}}D_{ij}^{(k)}d_{ij}^{(k)}-\sum_{j=n_i^{(k)}+1}^{p_i^{(k)}}D_{ij}^{(k)}\gamma_{ij}^{(k)}>0$$

$$V_i^{(k)}=\sum_{m=1}^{k-1}\left[\sum_{j=1}^{p^{(m)}}A_{ij}^{(k)}a_{ij}^{(k)} - \sum_{j=1}^{q_i^{(m)}}B_{ij}^{(k)}\beta_{ij}^{(k)}+\sum_{j=1}^{m^{(m)}}D_{ij}^{(k)}\delta_{ij}^{(k)}-\sum_{j=m_i^{(m)}+1}^{q_i^{(m)}}D_{ij}^{(k)}\delta_{ij}^{(k)}-\sum_{j=m_i^{(m}+1}^{q_i^{(m)}}D_{ij}^{(k)}d_{ij}^{(k)}\right.$$

$$\left.-\sum_{j=n_i^{(m)}+1}^{p_i^{(m)}}D_{ij}^{(k)}\gamma_{ij}^{(k)}\quad\right]>0 \tag{1.18}$$

and the $\bar{I}_k$ is defined by (1.9).

2. Theorem.

$$\text{If } \phi'(s_1,\cdots,s_r)=\bar{I}'[f(t_1',\cdots,t_r'):s_1,\cdots,s_r] \tag{2.1}$$

and

$$\phi^{(k)}(s_1,\cdots,s_r)=\bar{I}^{(k)}\left[\prod_{u=1}^{r}(t_u^{(k)})^{\rho_u^{(k-1)}}\phi^{(k-1)}\left(\frac{1}{t_1^{(k)}},\cdots\frac{1}{t_r^{(k)}}\right);s_1,\cdots,s_r\right] \tag{2.2}$$

then

$$\phi^{(N)}(s_1,\cdots,s_r)=\prod_{u=1}^{r}s_u^{-[\sum_{i=1}^{N-1}\rho_u^{(i)}+N-1]}\int_0^\infty\cdots\int_0^\infty \bar{I}_N(s_1t_1',\cdots,s_rt'r)f(t_1',\cdots,t_r')dt_1'\cdots dt_r' \tag{2.3}$$

where N is a positive integer greater than 1, the various integrals involved in the above theorem are absolutely convergent, the multivariable I-functions defined by Prathima et al [6]. occurring in the theorem satisfy their appropriate convergence and existence conditions and the conditions stated in (1.11) and (1.12) are satisfied.

Proof

Interpreting (2.2) for $k=2$ and using (1.8), we obtain

$$\phi''(s_1,\cdots,s_r)=\int_0^\infty\cdots\int_0^\infty\prod_{u=1}^{r}(t_u'')^{\rho_u'}\phi'\left(\frac{1}{t_1''},\cdots,\frac{1}{t_r''}\right)\bar{I}''(s_1t_1'',\cdots,s_rt_r'')\mathrm{d}t_1'',\cdots,\mathrm{d}t_r'' \tag{2.4}$$

Now substituting the value of ϕ' from (2.1) in (2.4), we have

$$\phi''(s_1,\cdots,s_r)=\int_0^\infty\cdots\int_0^\infty\prod_{u=1}^{r}(t''_u)^{\rho'_u}\quad \bar{I}''(s_1t''_1,\cdots,s_rt''_r)$$

$$\int_0^\infty\cdots\int_0^\infty \bar{I}'(t'_1(t''_1)^{-1},\cdots,t'_r(t''_r)^{-1})f(t'_1,\cdots,t'_r)\,\mathrm{d}t'_1,\cdots,\mathrm{d}t'_r\mathrm{d}t''_1,\cdots,\mathrm{d}t''_r \tag{2.5}$$

Now, changing the order of integration (which is premissible under the conditions stated with the theorem) and using (1.10) therein, we obtain

$$\phi''(s_1,\cdots,s_r)=\prod_{u=1}^{r}s_u^{-(\rho'_u+1)}\int_0^\infty\cdots\int_0^\infty \bar{I}_2(s_1t'_1,\cdots,s_rt'_r)f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r \tag{2.6}$$

so the result is true for $N=2$.

Let us assume that the result is true for $N=k$.Thus we have

$$\phi^{(k)}(s_1,\cdots,s_r)=\prod_{u=1}^{r}s_u^{-[\sum_{i=1}^{k-1}\rho_u^{(i)}+k-1]}\int_0^\infty\cdots\int_0^\infty \bar{I}_k(s_1t'_1,\cdots,s_rt'_r)f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r \tag{2.7}$$

From (2.2), we have

$$\phi^{(k+1)}(s_1,\cdots,s_r)=\bar{I}^{(k+1)}\left[\prod_{u=1}^{r}(t_u^{(k+1)\rho_u^{(k)}}\left(\frac{1}{t_1^{(k+1)}},\cdots,\frac{1}{t_r^{(k+1)}}\right):s_1,\cdots,s_r\right] \tag{2.8}$$

Now subsituting the value of $\phi^{(k)}$ from (2.7) in (2.8) and using (1.8) again, after algebraic manipulations, we get the following result.

$$\phi^{(k+1)}(s_1,\cdots,s_r)=\int_0^\infty\cdots\int_0^\infty(t_u^{(k+1)})^{[\sum_{i=1}^{k}\rho_u^{(i)}+k-1]}\ \bar{I}^{(k+1)}[s_1t_1^{(k+1)},\cdots,s_rt_r^{(k+1)}]$$

$$\int_0^\infty\cdots\int_0^\infty \bar{I}_k\left[t'_1(t_1^{(k+1)})^{-1},\cdots,t'_r(t_r^{(k+1)})^{-1}\right]f(t_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'r\mathrm{d}t_1^{(k+1)}\cdots\mathrm{d}t_r^{(k+1)} \tag{2.9}$$

Interchanging the order of integration in (2.9), which is premissible under the conditions stated the theorem and using (1.9), we get

$$\phi^{(k+1)}(s_1,\cdots s_r)=\prod_{u=1}^{r}s_u^{-[\sum_{i=1}^{k}\rho_u^{((i)}+k]}\int_0^\infty\cdots\int_0^\infty \bar{I}_{(k+1)}[s_1t'_1,\cdots,s_rt'_r]f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r \tag{2.10}$$

The result is true for $(k+1)$ also therefore by mathematical induction, the result is true for all positive integral value $N>1$.

3.Corollaries.

In this section, we shall see three corollaries.

The multivariable I-function defined by Prathima et al. [6] reduces in I-function of one variable defined by Rathie [7].

Corollary 1.

$$\left| \begin{matrix} (c_j^{(k)}, \gamma_j^{(k)}; C_j^{(k)})_{1,p^{(k)}} \\ \cdot \\ \cdot \\ (d_j^{(k)}, \delta_j^{(k)}; D_j^{(k)})_{1,q_1^{(k)}} \end{matrix} \right.$$

$$=$$

$$\frac{1}{2\pi\omega}\int_L \theta^{(k)}(\xi^{(k)})(z)^{\xi^{(k)}} d\xi \tag{3.1}$$

where

Throughout the paper we have used the following notations.

$$\left| \begin{matrix} C_k \\ \\ D_k \end{matrix} \right. \tag{3.3}$$

$$C_k = (c'_j, \gamma'_j; C'_j)_{1,n'}, \ \left[\left(c_j^{(l)} + \left[\sum_{i=1}^{l-1}\rho^{(i)} + l - 1\right]\gamma_j^{(l)}, \gamma_j^{(l)}; C_i^{(l)}\right)_{1,n^{(l)}}\right]_{l=2}^{k-1},$$

$$\left(c_j^{(k)} + \left[\sum_{i=1}^{k-1}\rho^{(i)} + k - 1\right]\gamma_j^{(k)}, \gamma_j^{(k)}; C_j^{(k)}\right)_{1,p^{(k)}},$$

$$(c'_j, \gamma'_j; C'_j)_{n'+1,p'}, \ \left[\left(c_j^{(l)} + \left[\sum_{i=1}^{l-1}\rho^{(i)} + l - 1\right]\gamma_j^{(l)}, \gamma_j^{(l)}; C_j^{(l)}\right)_{1+n^{(l)},p^{(l)}}\right]_{l=2}^{k-1} \tag{3.4}$$

$$D_k = (d'_j, \delta'_j; D'_j)_{1,m'}, \ \left[\left(d_j^{(l)} + \left[\sum_{i=1}^{l-1}\rho^{(i)} + l - 1\right]\delta_j^{(l)}, \delta_j^{(l)}; D_i^{(l)}\right)_{1,m^{(l)}}\right]_{l=2}^{k-1},$$

$$\left(d_j^{(k)} + \left[\sum_{i=1}^{k-1}\rho_1^{(i)} + k - 1\right]\delta_j^{(k)}\delta_j^{(k)}; D_j^{(k)}\right)_{1,q^{(k)}}, \ (d'_j, \delta_j; D'_j)_{n'+1,p'},$$

$$\left[\left(d_i^{(l)}+\left[\sum_{i=1}^{l-1}\rho_1^{(i)}+l-1\right]\delta_j^{(l)},\delta_j^{(l)};D_i^{(l)}\right)_{1,m^{(l)}}\right]_{l=2}^{k-1} \tag{3.5}$$

noneWe consider the multiple integrals in the left-hand side of (1.15), we replace the function I of r variables by the function I of one variable, nonethe corresponding integral noneconverges under the following conditions

$$U_i^{(k)}>0, V_i^{(k)}>0, |arg(s_i)|<\frac{1}{2}U_i^{(k)}\pi \tag{3.6}$$

$$\sum_{l=1}^{k-1}\max_{1\leq j\leq n^{(l)}} Re\left[C_j^{(l)}\frac{c_j^{(l)}-1}{\gamma_j^{(l)}}+\sum_{u=1}^{l-1}\rho^{(u)}+l-1\right]-\min_{1\leq j\leq m^{(k)}} D_j^{(k)}Re\frac{d_j^{(k)}}{\delta_j^{(k)}}< \ Re\left[\sum_{u=1}^{l-1}\rho_i^{(u)}+l-1\right]<$$

$$\sum_{l=1}^{k-1}\max_{1\leq j\leq n^{(l)}} Re\left[D_j^{(l)}\frac{d_j^{(l)}}{\delta_j^{(l)}}+\sum_{u=1}^{l-1}\rho^{(u)}+l-1\right]-\min_{1\leq j\leq m^{(k)}} C_j^{(k)}Re\frac{c_j^{(k)}-1}{\gamma_j^{(k)}} \tag{3.7}$$

where

$$U^{(k)}=\sum_{j=1}^{m^{(k)}}A_j^{(k)}-\sum_{j=m^{(k)}+1}^{q^{(k)}}A_j^{(k)}+\sum_{j=1}^{n^{(k)}}B_j^{(k)}-\sum_{j=n^{(k)}+1}^{p^{(k)}}B_j^{(k)}>0 \tag{3.8}$$

$$V^{(k)}=\sum_{l=1}^{k-1}\left[\sum_{j=1}^{m^{(l)}}A_j^{(k)}-\sum_{j=m^{(l)}+1}^{q^{(l)}}A_j^{(k)}-\sum_{j=1}^{n^{(l)}}B_j^{(k)}-\sum_{j=n^{(l)}+1}^{p^{(l)}}B_j^{(k)}\ \right]>0 \tag{3.9}$$

The following corollaries have been studied by Gupta et al. [5].

If we reduce the multidimensional $\bar{I}$-transforms occurring in (2.1) and (2.2) to multidimensional Laplace transforms, we obtain the following result :

Corollary 2.

If $\phi'(s_1,\cdots,s_r)=L[f(t_1',\cdots,t_r'):s_1,\cdots,s_r]$

and

$$\phi^{(k)}(s_1,\cdots,s_r)=L\left[\prod_{u=1}^{r}(t_u^{(k)})^{\rho_u^{(k-1)}}\phi^{(k-1)}\left(\frac{1}{t_1^{(k)}},\cdots,\frac{1}{t_r^{(k)}}\right);s_1,\cdots,s_r\right]$$

$$\forall k\in\{2,\cdots,N\}$$

then

$$\phi^{(N)}(s_1,\cdots,s_r)=\prod_{u=1}^{r}s_u^{\left[\sum_{i=1}^{N-1}\rho_u^{(i)}+N-1\right]}\int_0^\infty\cdots\int_0^\infty\prod_{u=1}^{r}H_{0,N}^{N,0}\left(\ s_ut'_u\ \middle|\ (0,1),\left[\sum_{i=1}^{l-1}\rho_u^{(i)}+l-1,1\right]_{l=2,N}\ \right)$$

$$f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r \tag{3.10}$$

Again, if we reduce the multivariable I-function of Prathima et al. [6] involved in the main theorem into product of r- generalized Bessel function ($J_\lambda^{(u)}$) [8, p.19, eq. (2.6.10)], we obtain

Corollary 3.

$$\text{If } \phi'(s_1,\cdots,s_r)=\int_0^\infty\cdots\int_0^\infty\prod_{u=1}^{r}J_{\lambda_u'}^{\upsilon'_u}[s_ut'_u]\,f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r \tag{3.11}$$

and

$$\phi^{(k)}(s_1,\cdots,s_r)=\int_0^\infty\cdots\int_0^\infty\prod_{u=1}^{r}(t_u^{(k)})^{\rho_u^{(k-1)}}J_{\lambda_u^{(k)}}^{\upsilon_u^{(k)}}[s_ut_u^{(k)}]\phi^{(k-1)}\left(\frac{1}{t_1^{(k)}},\cdots,\frac{1}{t_r^{(k)}}\right)\mathrm{d}t_1^{(k)}\cdots\mathrm{d}t_r^{(k)} \tag{3.12}$$

$$\forall k\in\{2,\cdots,N\}$$

then

$$\left|\begin{matrix}-\\ \mathrm{d}_u\end{matrix}\right.$$

$$f(t'_1,\cdots,t'_r)\mathrm{d}t'_1\cdots\mathrm{d}t'_r$$

(3.13)

where

$$d_u=(0,1),\left[\left(\sum_{i=1}^{l-1}(\rho_u^{(i)}+l-1\right),1\right]_{2,N},(-\lambda'_u,\upsilon'_u),\left[-\lambda_u^{(l)}+\left(\sum_{i=1}^{l-1}\rho_u^{(i)}+l-1\right)\upsilon_u^{(l)},u^{(l)}\right]_{2,N} \tag{3.14}$$

and the validity conditions are directly obtained from the main theorem are assumed to be satisfied.

4. Conclusion.

The importance of our all the results lies in their manifold generality. By specializing the various parameters as well as variables in the multivariable I-function defined by Prathima et al. [6], we obtain a large number of results involving remarkably wide variety of useful special functions (or product of such special functions) which are expressible in terms of I-function, H-function, Meijer's G-function, E-function and hypergeometric function of one and several variables ,etc. Therefore we can get a large number of chains involving the special functions by using the multidimensional integral transformations.

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