Cn$\mathcal{I}$g-Continuous Maps in Nano Ideal Topological Spaces

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Abstract

The aim of in this paper, we introduced n$\mathcal{I}$g-interior, n$\mathcal{I}$g-closure and study some of its basic properties. we introduced and studied n$\mathcal{I}$g-continuous map, n$\mathcal{I}$g-irresolute map and study their properties in nano ideal topological spaces.

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1 Introduction and preliminaries

Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on U, where $\mathcal{N} = \tau_R(X) and (.)^*_n : \wp$(U)→$\wp$(U) ($\wp$(U) is the set of all subsets of U) [5, 6]. For a subset A ⊆U, A$^*_n(\mathcal{I}, \mathcal{N})$={x∈U : G$_n$∩ A $\notin \mathcal{I}$, for every G$_n$∈ G$_n(x)\}, where \{G_n = \{G_n|$ x∈ G$_n, G_n \in \mathcal{N}\}$is called the nano local function(briefly n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A$^*_n$ for A$^*_n$ ($\mathcal{I}$, $\mathcal{N}$).

Nano ideal generalized closed sets were introduced and studied by Parimala et al [6]. In this paper, we first introduced n$\mathcal{I}$g-interior, n$\mathcal{I}$g-closure and study some of its basic properties. we introduced and studied n$\mathcal{I}$g-continuous map, n$\mathcal{I}$g-irresolute map. We also discuss some properties of n$\mathcal{I}$g-continuous in nano ideal topological spaces.

Definition 1.1 *[4]*

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.*

   $Thatis, L_R(X) = \bigcup x \epsilon U \{R(X) : R(X) \subseteq X\}$ *where R(x) denotes the equivalence class determined by X.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by* $U_R(X)$.
   $That is, U_R(X) = \bigcup x \epsilon U\{R(X) : R(X) \cap X \neq \phi\}$.

3. *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by* $B_R(X). That is, B_R(X) = U_R(X) - L_R(X)$.

Property 1.2 *[4]*

If (U, R) is an approximation space and X, Y $\subseteq$ *U, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$.
2. $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
7. $L_R(X) \subseteq L_R(Y) and U_R(X) \subseteq U_R(Y) whenever X \subseteq Y$.
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
9. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.
10. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 1.3 *[4] Let U be an universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi , L_R(X), U_R(X), B_R(X)\} where X$ *U. Then by property 1.2,* $\tau_R(X)$ *satisfies the following axioms*

1. $U , \phi \ \epsilon \ \tau_R(X)$.
2. *The union of the elements of any sub-collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.
3. *The intersection of the elements of any finite sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

   Then $\tau_R(X)$ *is called the Nano topology on U with respect to X.*

   The space (K, $\tau_R(X)$*) is the Nano Nano topological space. The elements of are called Nano open sets.*

Definition 1.4 *[4]*

If (U, $\tau_R(X)$ *is the Nano topological space with respect to X where X* $\subseteq$ *U and if M* $\subseteq$ *U, then*

1. *The Nano interior of the set M is defined as the union of all Nano open subsets contained in M and it is denoted by NInte(M). That is, NInte(M) is the largest Nano open subset of M.*

2. *The Nano closure of the set M is defined as the intersection of all Nano closed sets containing M and it is denoted by NClo(M). That is, NClo(M) is the smallest Nano closed set containing M.*

Theorem 1.5 *[5, 6] Let $(U, \mathcal{N})$ be a nano topological space with ideal $\mathcal{I}$, $\mathcal{I}'$ on U and A, B be subsets of U. Then*

1. $A \subseteq B \Rightarrow A^*_n \subseteq B^*_n$.

2. $\mathcal{I} \subseteq \mathcal{I}' \Rightarrow A^*_n\ (\mathcal{I}') \subseteq A^*_n\ (\mathcal{I})$.

3. A^*_n=n-cl(A^*_n) $\subseteq$ n-cl(A) (A^*_n *is a nano closed subset of* n-cl(A)).

4. $(A^*_n)^*_n \subseteq A^*_n$.

5. $A^*_n \cup B^*_n = (A \cup B)^*_n$

6. $A^*_n - B^*_n = (A - B)^*_n - B^*_n \subseteq (A - B)^*_n$.

7. $V \in \mathcal{N} \Rightarrow V \cap A^*_n = V \cap (V \cap A)^*_n \subseteq (V \cap A)^*_n$ *and*

8. $J \in \mathcal{I} \Rightarrow (A \cup J)^*_n = A^*_n = (A - J)^*_n$

Lemma 1.6 *[5, 6] Let $(U, \mathcal{N}, \mathcal{I})$ be an nano topological space with an ideal $\mathcal{I}$ and $A \subseteq A^*_n$, then A^*_n=n-cl(A^*_n)=n-cl(A)*

Definition 1.7 *[5, 6] Let $(U, \mathcal{N})$ be an nano topological space with an ideal $\mathcal{I}$ on U. The set operator n-cl* is called a nano $\star$ -closure and is defined as n-cl*(A)= $A \cup A^*_n$ for $A \subseteq X$.*

Theorem 1.8 *[5, 6] The set operator n-cl* satisfies the following conditions:*

1. $A \subseteq$ n-cl*(A).

2. n-cl$^*(\phi) = \phi$ *and* n-cl$^*(U) = U$.

3. *If* $A \subseteq B$, *then* n-cl$^*(A) \subseteq$ n-cl$^*(B)$.

4. n-cl$^*(A) \cup$ n-cl$^*(B)$ = n-cl$^*(A \cup B)$

5. n-cl*(n-cl$^*(A)$) =n-cl$^*(A)$

Definition 1.9 *[5, 6] A subset A of a nano ideal topological space $(U, \mathcal{N}, \mathcal{I})$ is n$\star$-closed if $A^*_n \subseteq A$.*

Definition 1.10 *[5, 6] A subset A of an nano ideal topological space $(O, \mathcal{N}, \mathcal{I})$ is said to be*

1. *nano-$\mathcal{I}$-generalized closed (briefly, $n\mathcal{I}g$-closed if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n-open.*
2. *$n\mathcal{I}g$-open if its complement is $n\mathcal{I}g$-closed.*

Remark 1.11 *The collection of all $n\mathcal{I}g$-closed(resp. $n\mathcal{I}g$-open) sets is denoted by $n\mathcal{I}g$-c($\mathcal{N}$) (resp.$n\mathcal{I}g$-o($\mathcal{N}$)).*

Definition 1.12 *[7] A subset A of a nano ideal topolgical space $(U, \mathcal{N}, \mathcal{I})$ is said to be nano-$\mathcal{I}$-open (briefly, $n\mathcal{I}$-open) if $A \subseteq n\text{-}int(A_n^*)$.*

Remark 1.13

1. *Every n-closed set is $n\star$-closed but not conversely [1].*
2. *Every $n\star$-closed set is $n\mathcal{I}g$-closed but not conversely [6].*

Definition 1.14 *A map f: $(K, \mathcal{N}, \mathcal{I}) \to (L, \mathcal{N}')$ is said to be $n\star$-continuous [3] if $f^{-1}(A)$ is $n\star$-closed in $(K, \mathcal{N}, \mathcal{I})$ for every n-closed set A of $(L, \mathcal{N}')$.*

2 n$\mathcal{I}$g-INTERIOR AND n$\mathcal{I}$g-CLOSURE

Definition 2.1 *For any $M \subseteq O$, $n\mathcal{I}$-int(M) is defined as the union of all $n\mathcal{I}$-open sets contained in M. i.e., $n\mathcal{I}$-int(M) $= \cup \{G : G \subseteq M$ and G is $n\mathcal{I}$-open$\}$.*

Definition 2.2 *For any $M \subseteq O$, $n\mathcal{I}g$-int(M) is defined as the union of all $n\mathcal{I}g$-open sets contained in M. i.e., $n\mathcal{I}g$-int(M) $= \cup \{G : G \subseteq M$ and G is $n\mathcal{I}g$-open$\}$.*

Lemma 2.3 *For any $M \subseteq O$, $n\mathcal{I}$-int(M) $\subseteq$ $n\mathcal{I}g$-int(M) $\subseteq M$.*

The proof follows from Definitions 2.1 and 2.2.

The following two Propositions are easy consequences from definitions.

Proposition 2.4 *For any $M \subseteq O$, the following holds.*

1. *$n\mathcal{I}g$-int(M) is the largest $n\mathcal{I}g$-open set contained in M.*
2. *M is $n\mathcal{I}g$-open if and only if $n\mathcal{I}g$-int(M) $= M$.*

Proposition 2.5 *For any subsets M and P of $(O, \mathcal{N}, \mathcal{I})$, the following holds.*

1. *$n\mathcal{I}g$-int$(M \cap P) = n\mathcal{I}g$-int$(M) \cap n\mathcal{I}g$-int$(P)$.*

2. *$n\mathcal{I}g$-int$(M \cup P) \supseteq n\mathcal{I}g$-int$(M) \cup n\mathcal{I}g$-int$(P)$.*

3. *If $M \subseteq P$, then $n\mathcal{I}g$-int$(M) \subseteq n\mathcal{I}g$-int$(P)$.*

4. *$n\mathcal{I}g$-int$(O) = O$ and $n\mathcal{I}g$-int$(\phi) = \phi$.*

Definition 2.6 *For every set $M \subseteq O$, we define the $n\mathcal{I}g$-closure of M to be the intersection of all $n\mathcal{I}g$-closed sets containing M. i.e., $n\mathcal{I}g$-cl$^*(M) = \cap \{F : M \subseteq F \in n\mathcal{I}gc(\mathcal{N})$.*

Lemma 2.7 *For any $M \subseteq O$, $M \subseteq n\mathcal{I}g$-cl$^*(M) \subseteq n$-cl$^*(M)$.*

The proof follows from Remark 1.13(2).

Remark 2.8 *Both containment relations in Lemma 2.7 may be proper as seen from the following example.*

Example 2.9 *Let $O = \{a, b, c\}$, with $O/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then the Nano topology $\mathcal{N} = \{\phi, \{a\}, O\}$ and $\mathcal{I} = \{\emptyset\}$. Then $n\mathcal{I}g$-closed sets are $\emptyset$, O, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$. Let $A = \{a, b\}$. Here $n\mathcal{I}g$-cl$^*(\{a, b\}) = \{a, b\}$, n-cl$^*(\{a, b\}) = O$ and so $A \subseteq n\mathcal{I}g$-cl$^*(A) \subseteq n$-cl$^*(A)$.*

The following two Propositions are easy consequences from definitions.

Proposition 2.10 *For any $M \subseteq O$, the following holds.*

1. *$n\mathcal{I}g$-cl$^*(M)$ is the smallest $n\mathcal{I}g$-closed set containing M.*

2. *M is $n\mathcal{I}g$-closed if and only if $n\mathcal{I}g$-cl$^*(M) = M$.*

Proposition 2.11 *For any two subsets M and P of $(O, \mathcal{N}, \mathcal{I})$, the following holds.*

1. *If $M \subseteq P$, then $n\mathcal{I}g$-cl$^*(M) \subseteq n\mathcal{I}g$-cl$^*(P)$.*

2. *$n\mathcal{I}g$-cl$^*(M \cap P) \subseteq n\mathcal{I}g$-cl$^*(M) \cap n\mathcal{I}g$-cl$^*(P)$.*

Proposition 2.12 *Let M be a subset of a space O, then the following are true.*

1. *$(n\mathcal{I}g$-int$(M))^c = n\mathcal{I}g$-cl$^*(M^c)$.*

2. *$n\mathcal{I}g$-int$(M) = (n\mathcal{I}g$-cl$^*(M^c))^c$.*

3. *$n\mathcal{I}g$-cl$^*(M) = (n\mathcal{I}g$-int$(M^c))^c$.*

Proof

1. *Clearly follows from definitions.*

2. *Follows by taking complements in (1).*

3. *Follows by replacing M by M^c in (1).*

3 n$\mathcal{I}$g-CONTINUOUS MAPS

Definition 3.1 *[2] A map f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) is called n$\mathcal{I}$g-continuous if $f^{-1}(V)$ is a n$\mathcal{I}$g-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set V of (P, $\mathcal{N}'$).*

Proposition 3.2 *Every ⋆-continuous is n$\mathcal{I}$g-continuous but not conversely.*

The proof follows from Result 1.13(2).

Example 3.3 *Let O, $\mathcal{N}$ and $\mathcal{I}$ be defined as Example 2.9. Then n⋆-closed sets are ∅, O, {b, c}. Let P = {a, b, c} with with P/ R= {{c}, {a, b}} and X= {b, c}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {c}, {a, b}, P} and $\mathcal{J}$ = {∅}. Then n$\mathcal{I}$g-closed sets are ∅, O, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) be the identity map. Then f is n$\mathcal{I}$g-continuous but not n⋆-continuous, since f^{-1}({a, b})= {a, b} is not n⋆-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Remark 3.4 *The composition of two n$\mathcal{I}$g-continuous maps need not be n$\mathcal{I}$g-continuous and this is shown from the following example.*

Example 3.5 *Let O, $\mathcal{N}$ and $\mathcal{I}$ be as in Example 2.9. Let P = {a, b, c}, with P/ R= {{c}, {a, b}, {b, a}} and X= {a, b}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {a, b}, P} and $\mathcal{I}$={∅, {a}}. Then n$\mathcal{I}$g-closed sets are ∅, P, {a}, {c}, {a, c}, {b, c}. Let Q = {a, b, c} with Q/ R= {{c}, {a, b}} and X= {b, c}. Then the Nano topology $\mathcal{N}'_*$= {ϕ, {c}, {a, b}, Q} and $\mathcal{K}$ = {ϕ, {Q}}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) by f(a) = a, f(b) = c and f(c) = b. Define g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) by g(a) =c, g(b)= b and g(c)= a. Clearly f and g are n$\mathcal{I}$g-continuous but their g ∘ f : (O, $\mathcal{N}$, $\mathcal{I}$) → (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is not n$\mathcal{I}$g-continuous, because V = {c} is n-closed in (Q, $\mathcal{N}'_*$) but (g ∘ f $^{-1}$({c})= $f^{-1}(g^{-1}$({c})) = f^{-1}({a}) = {a}, which is not n$\mathcal{I}$g-closed in (O, $\mathcal{N}$, $\mathcal{I}$) .*

Proposition 3.6 *A map f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) is n$\mathcal{I}$g-continuous if and only if $f^{-1}(U)$ is n$\mathcal{I}$g-open in (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set U in (P, $\mathcal{N}'$).*

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) be n$\mathcal{I}$g-continuous and U be an n-open set in (P, $\mathcal{N}'$). Then U^c is n-closed in (P, $\mathcal{N}'$) and since f is n$\mathcal{I}$g-continuous, $f^{-1}(U^c)$ is n$\mathcal{I}$g-closed in (O, $\mathcal{N}$, $\mathcal{I}$) . But $f^{-1}(U^c)$ = $f^{-1}((U))^c$ and so $f^{-1}(U)$ is n$\mathcal{I}$g-open *in (O, $\mathcal{N}$, $\mathcal{I}$).*

Conversely, assume that $f^{-1}(U)$ is n$\mathcal{I}$g-open in (O, $\mathcal{N}$, $\mathcal{I}$) for each n-open set U in (P, $\mathcal{N}'$). Let F be a n-closed set in (P, $\mathcal{N}'$). Then F^c is n-open in (P, $\mathcal{N}'$) and by assumption, $f^{-1}(F^c)$ is n$\mathcal{I}$g-open in (O, $\mathcal{N}$, $\mathcal{I}$). Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have $f^{-1}(F)$ is n-closed in (O, $\mathcal{N}$, $\mathcal{I}$) and so f is n$\mathcal{I}$g-continuous.

We introduce the following definition

Definition 3.7 *A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called n$\mathcal{I}$g-irresolute if $f^{-1}(V)$ is a n$\mathcal{I}$g-closed set of (O, $\mathcal{N}$, $\mathcal{I}$)for every n$\mathcal{I}$g-closed set V of (P, $\mathcal{N}'$, $\mathcal{J}$).*

Theorem 3.8 *Every n$\mathcal{I}$g-irresolute map is n$\mathcal{I}$g-continuous but not conversely.*

Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a n$\mathcal{I}$g-irresolute map. Let V be a n-closed set of (P, $\mathcal{N}'$). Then by the Result 1.13 (1) and (2), V is n$\mathcal{I}$g-closed. Since f is n$\mathcal{I}$g-irresolute, then $f^{-1}(V)$ is a n$\mathcal{I}$g-closed set of (O, $\mathcal{N}$, $\mathcal{I}$). Therefore f is n$\mathcal{I}$g-continuous.

Example 3.9 *Let O, $\mathcal{N}$ with $\mathcal{I}$ be as in the Example 2.9. Let P, $\mathcal{N}'$ with $\mathcal{J}$ be as in the Example 3.3. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. It is clear that $\{a\}$ is n$\mathcal{I}$g-closed set of (P, $\mathcal{N}'$, $\mathcal{J}$) but $f^{-1}(\{a\})$ =$\{a\}$ is not a n$\mathcal{I}$g-closed set of (O, $\mathcal{N}$, $\mathcal{I}$). Thus f is not n$\mathcal{I}$g-irresolute map. However f is n$\mathcal{I}$g-continuous map.*

Theorem 3.10 *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) and g : (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) be any two maps. Then*

1. *$g \circ f$ is n$\mathcal{I}$g-continuous if g is n$\star$-continuous and f is n$\mathcal{I}$g-continuous.*
2. *$g \circ f$ is n$\mathcal{I}$g-irresolute if both f and g are n$\mathcal{I}$g-irresolute.*
3. *$g \circ f$ is n$\mathcal{I}$g-continuous if g is n$\mathcal{I}$g-continuous and f is n$\mathcal{I}$g-irresolute.*

Omitted.

Definition 3.11 *Let (O, $\mathcal{N}$, $\mathcal{I}$) be a nano ideal topological space. Let o be a point of O and G be a subset of O. Then G is called an n$\mathcal{I}$g-neighbourhood of o (briefly, n$\mathcal{I}$g-nbhd of o) in O if there exists an n$\mathcal{I}$g-open set S of O such that o $\in S \subseteq G$.*

Proposition 3.12 *Let M be a subset of (O, $\mathcal{N}$, $\mathcal{I}$). Then o $\in$ n$\mathcal{I}$g-cl*(M) if and only if for any n$\mathcal{I}$g-nbhd G_o of o in (O, $\mathcal{N}$, $\mathcal{I}$), $M \cap G_o \neq \phi$.*

Necessity. Assume o $\in$ n$\mathcal{I}$g-cl(M). Suppose that there is an n$\mathcal{I}$g-nbhd G of the point o in (O, $\mathcal{N}$, $\mathcal{I}$) such that $G \cap M = \phi$. Since G is n$\mathcal{I}$g-nbhd of o in (O, $\mathcal{N}$, $\mathcal{I}$), by Definition 3.11, there exists an n$\mathcal{I}$g-open set S_o such that o $\in S_o \subseteq G$. Therefore, we have $S_o \cap M = \phi$ and so $M \subseteq (S_o)^c$. Since $(S_o)^c$ is an n$\mathcal{I}$g-closed set containing M, we have by*

Definition 2.6, $n\mathcal{I}g$-$cl^*(M) \subseteq (S_o)^c$ and therefore $o \notin n\mathcal{I}g$-$cl^*(M)$, which is a contradiction.

Sufficiency. Assume for each $n\mathcal{I}g$-nbhd G_o of o in $(O, \mathcal{N}, \mathcal{I})$, $M \cap G_k \neq \phi$. Suppose that $o \notin n\mathcal{I}g$-$cl^*(M)$. Then by Definition 2.6, there exists an $n\mathcal{I}g$-closed set F of $(O, \mathcal{N}, \mathcal{I})$ such that $M \subseteq F$ and $o \notin F$. Thus $o \in F^c$ and F^c is $n\mathcal{I}g$-open in $(O, \mathcal{N}, \mathcal{I})$ and hence F^c is a $n\mathcal{I}g$-nbhd of o in $(O, \mathcal{N}, \mathcal{I})$. But $M \cap F^c = \phi$, which is a contradiction.

In the next theorem we explore certain characterizations of $n\mathcal{I}g$-continuous maps.

***Theorem 3.13** Let f: $(O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}')$ be a map. Then the following statements are equivalent.*

1. *The map f is $n\mathcal{I}g$-continuous.*
2. *The inverse of each n-open set is $n\mathcal{I}g$-open.*
3. *For each point o in $(O, \mathcal{N}, \mathcal{I})$ and each n-open set V in $(P, \mathcal{N}')$ with $f(o) \in V$, there is an $n\mathcal{I}g$-open set U in $(O, \mathcal{N}, \mathcal{I})$ such that $o \in U$, $f(U) \subseteq V$.*
4. *The inverse of each n-closed set is $n\mathcal{I}g$-closed.*
5. *For each o in $(O, \mathcal{N}, \mathcal{I})$, the inverse of every neighbourhood of $f(o)$ is an $n\mathcal{I}g$-nbhd of o.*
6. *For each o in $(O, \mathcal{N}, \mathcal{I})$ and each neighbourhood N of $f(o)$, there is an $n\mathcal{I}g$-nbhd G of o such that $f(G) \subseteq N$.*
7. *For each subset A of $(O, \mathcal{N}, \mathcal{I})$, $f(n\mathcal{I}g\text{-}cl^*(A)) \subseteq cl^*(f(A))$.*
8. *For each subset B of $(P, \mathcal{N}')$, $n\mathcal{I}g\text{-}cl^*(f^{-1}(B)) \subseteq f^{-1}(cl^*(B))$.*

(1) $\Leftrightarrow$ (2). This follows from Proposition 3.6.

(1) $\Leftrightarrow$ (3). Suppose that (3) holds and let V be an n-open set in $(P, \mathcal{N}')$ and let $o \in f^{-1}(V)$. Then $f(o) \in V$ and thus there exists an $n\mathcal{I}g$-open set U_o such that $o \in U_o$ and $f(U_o) \subseteq V$. Now, $o \in U_o \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup_{o \in f^{-1}(V)} U_o$. By assumption, $f^{-1}(V)$ is $n\mathcal{I}g$-open in $(O, \mathcal{N}, \mathcal{I})$ and therefore f is $n\mathcal{I}g$-continuous.

Conversely, Suppose that (1) holds and let $f(o) \in V$. Then $o \in f^{-1}(V) \in n\mathcal{I}go(\mathcal{N})$, since f is $n\mathcal{I}g$-continuous. Let $U = f^{-1}(V)$. Then $o \in U$ and $f(U) \subseteq V$.

(2) $\Leftrightarrow$ (4). This result follows from the fact if A is a subset of $(P, \mathcal{N}')$, then $f^{-1}(A^c) = (f^{-1}(A))^c$.

(2) $\Leftrightarrow$ (5). For o in $(O, \mathcal{N}, \mathcal{I})$, let N be a neighbourhood of $f(o)$. Then there exists an n-open set U in $(P, \mathcal{N}')$ such that $f(o) \in U \subseteq N$. Consequently, $f^{-1}(U)$ is an $n\mathcal{I}g$-open set in $(O, \mathcal{N}, \mathcal{I})$ and $o \in f^{-1}(U) \subseteq f^{-1}(N)$. Thus $f^{-1}(N)$ is an $n\mathcal{I}g$-nbhd of o.

(5) $\Leftrightarrow$ (6). Let $o \in O$ and let N be a neighbourhood of $f(o)$. Then by assumption, $G = f^{-1}(N)$ is an $n\mathcal{I}g$-nbhd of o and $f(G) = f(f^{-1}(N)) \subseteq N$.

(6) $\Leftrightarrow$ (3). For o in $(O, \mathcal{N}, \mathcal{I})$, let V be an n-open set containing $f(o)$. Then V is a neighborhood of $f(o)$. So by assumption, there exists an $n\mathcal{I}g$-nbhd G of o such that $f(G) \subseteq V$. Hence there exists an $n\mathcal{I}g$-open set U in $(O, \mathcal{N}, \mathcal{I})$ such that $o \in U \subseteq G$ and so $f(U) \subseteq f(G) \subseteq V$.

(7)⇔ (4). Suppose that (4) holds and let A be a subset of $(O, \mathcal{N}, \mathcal{I})$. *Since* $A \subseteq f^{-1}(A)$, *we have* $A \subseteq f^{-1}(cl^*(f(A)))$. *Since* $cl^*(f(A))$ *is a n-closed set in* $(P, \mathcal{N}')$, *by assumption* $f^{-1}(cl^*(f(A)))$ *is an* $n\mathcal{I}g$*-closed set containing A. Consequently,* $n\mathcal{I}g\text{-}cl^*(A) \subseteq f^{-1}(cl^*(f(A)))$. *Thus* $f(n\mathcal{I}g\text{-}cl^*(A)) \subseteq f(f^{-1}(cl^*(f(A)))) \subseteq cl^*(f(A))$.

Conversely, suppose that (7) holds for any subset A of $(O, \mathcal{N}, \mathcal{I})$. *Let F be a n-closed subset of* $(P, \mathcal{N}')$. *Then by assumption,* $f(n\mathcal{I}g\text{-}cl^*(f^{-1}(F))) \subseteq cl^*(f(f^{-1}(F))) \subseteq cl^*(F) = F$. *i.e.,* $n\mathcal{I}g\text{-}cl^*(f^{-1}(F)) \subseteq f^{-1}(F)$ *and so* $f^{-1}(F)$ *is* $n\mathcal{I}g$*-closed.*

(7) ⇔ (8). Suppose that (7) holds and B be any subset of $(P, \mathcal{N}')$. *Then replacing A by* $f^{-1}(B)$ *in (7), we obtain* $f(n\mathcal{I}g\text{-}cl^*(f^{-1}(B))) \subseteq cl^*(f(f^{-1}(B))) \subseteq cl^*(B)$. *i.e.,* $n\mathcal{I}g\text{-}cl^*(f^{-1}(B)) \subseteq f^{-1}cl^*(B)$.

Conversely, suppose that (8) holds. Let $B = f(A)$ *where A is a subset of* $(O, \mathcal{N}, \mathcal{I})$. *Then we have,* $n\mathcal{I}g\text{-}cl^*(A) \subseteq n\mathcal{I}g\text{-}cl^*(f^{-1}(B)) \subseteq f^{-1}(cl^*(f(A))$ *and so* $f(n\mathcal{I}g\text{-}cl^*(A)) \subseteq cl^*(f(A))$.

This completes the proof of the theorem.

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