Discovery of ambiguity in the typical process of integration in Integral calculus

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Abstract

The fundamental concept behind the integration operation in Integral calculus has been revisited. Information gathered from traditional literature search regarding the typical process of integration adopted by various authors to solve problem or in going through theoretical discussions has been first shared with and subsequently put into context to the fundamental concept of integration operation in Integral calculus to find that there exists ambiguity in the procedural steps of performing such integration process. With a view to getting rid of the ambiguous procedure inherent in the process of integration, the unambiguous procedural steps to be followed for solving such a problem as well as in going through each of the relevant part of those theoretical discussions have been finally offered.

Key Words: Integral calculus; Differential calculus; Function; Notation for integral; Notation for differential of a variable; Notation for the operator of integration.

Introduction

In order to bring preciseness and sophistication in the study of Science and Engineering, it is essential to get rid of the ambiguous concepts and procedures prevailing so far in the relevant traditional literature and replace those ambiguous concepts and procedures by the unambiguous ones. Several attempts have been made so far by Bhattacharjee (2012, 2017, 2018, 2020) in the aforesaid context. The author in (Bhattacharjee, 2012) considered the ambiguous concept of negative distance (which does not have any compliance with the real world situation) prevailing so far in the long-running definitions of trigonometric ratios with simultaneous incorporation of the unambiguous definitions of more realistic trigonometric ratios by making use of vector algebra. That the traditional definitions of angle of diffraction and glancing angle are ambiguous have been pointed out in (Bhattacharjee, 2017) along with disclosing the corresponding unambiguous definition of each of the aforesaid two angles. The author in (Bhattacharjee, 2018) reported a discovery of misleading graph titles at many places of the traditional scientific literature and proposed a suggestion for writing down an unambiguous title of a graph after completion of graph drawing. Discovery of ambiguity in the traditional procedure of mathematical handling of physical quantities has been recently reported in (Bhattacharjee, 2020) along with offering the unambiguous procedure to be adopted in the said context.

A further step forward in the aforesaid context is the present contribution in which the discovery of ambiguity in the traditional procedure of performing integration at many places of the long-running literature has been disclosed. It may be noted that the study of science and engineering is based on widely accepted notations and conventions. For a systematic study one has to follow the long-used unambiguous notations and conventions. The fundamental concept of notation used to denote the integration operation of a function in Integral calculus is well-known. But it has been detected in the present study that many traditional literature (Dasgupta, 1997; De, 1998; Kachhava, 1990; Paul, 2019; Zhang & Li, 2018) do not take proper care of the aforesaid fundamental concept of notation of the integration operation. As a result, there exists ambiguity in the procedural steps of integration adopted in the solution of the example problem and in each part of the theoretical discussions considered in this paper (Dasgupta, 1997; De, 1998; Kachhava, 1990; Paul, 2019; Zhang & Li, 2018).

The standard notation for integration operation has been presented first. Then some materials that resulted from the search of traditional literature in connection with the performance of integration process have been incorporated. An examination of the approach adopted for integration operation in the solution of the example problem and in each part of theoretical discussions considered (Dasgupta, 1997; De, 1998; Kachhava, 1990; Paul, 2019; Zhang & Li, 2018) has been made subsequently to find the existence of ambiguity in the approach adopted in all those cases (Dasgupta, 1997; De, 1998; Kachhava, 1990; Paul, 2019; Zhang & Li, 2018). Finally, unambiguous procedural steps to be followed in regard to the performance of the integration process for each of those cases have been offered.

Revisiting the fundamental concept regarding notation of the process of integration in Integral calculus

This section considers incorporation of the fundamental fact in regard to the notation of the process of integration in Integral calculus that resulted from standard literature search.

The following quoted lines in relation to the discussion of 'Indefinite Integral Notations' exist in (Zill& Wright, 2009, p. 269).

"**Indefinite Integral Notations**

For convenience, let us introduce a notation for an anti-derivative of a function. If $F^{/}(x) = f(x)$, we shall represent the most general anti-derivative of f by

$$\int f(x)\, dx = F(x) + C$$

The symbol ∫ was introduced by Leibniz and is called an **Integral sign**. The notation ∫ f(x) dx is called the **Indefinite Integral** of f(x) with respect to x. The function f(x) is called the **Integrand**. The process of finding an anti-derivative is called **anti-differentiation** or **integration**. Just as $\frac{d}{dx}(\ )$ denotes the operator of differentiation of ( ) with respect to x, the symbolism ∫ ( ) dx denotes the operation of integration of ( ) with respect to x."

Furthermore, attention is being drawn to the following quoted lines prevailing in (De, 1998, p. 220). "... ; to signify the integration operation the ***Integral sign*** ∫ is written before the given function and the differential is written after the given function to indicate that is the ***variable of integration***."

It thus follows from above that to denote the 'process of integration' of the function f(x) with respect to the variable x, simultaneous use of both the symbols '∫' and 'dx' are needed. In other words, to indicate the 'process of integration' of the function f(x) with respect to the variable x, the function f(x) is to be inserted in between the two symbols '∫' and 'dx'.

Information gathered from traditional literature search regarding typical process of integration

In this section, information gathered from search of relevant materials has been incorporated from the view point of general interest and subsequent examination.

(i) The following quoted lines indicate the procedure followed by the author in (De,1998, p. 380) to solve the differential equation of Example 2.

" Example 2. Solve $x\frac{dy}{dx} + y^2 = 4$.

Solution: $x\frac{dy}{dx} + y^2 = 4$

or, $x\frac{dy}{dx} = 4 - y^2$

or, $\frac{dy}{4 - y2} = \frac{dx}{x}$

or, $\int \frac{dy}{4 - y2} = \int \frac{dx}{x} + k$

or, $\frac{1}{2.\,2}\log\left|\frac{2+y}{2-y}\right| = \log|x| + \frac{1}{4}\log c$ "

(ii) The quoted lines below indicate a part of the procedure followed by the author in (Dasgupta, 1997, p. 16) for the Calculus based derivation of an equation of rectilinear motion.

"If *ds* be the small distance travelled by the particle in an infinitesimally short interval of time *dt* during which the velocity *v* can be supposed to be constant, then, $ds = v\,.\,dt = (u + ft)\,dt$.

Integrating, $\int ds = \int (u + ft)dt = \cdots$, or, $s = ut + \frac{1}{2}ft^{2} + C_2$"

(iii) The following quoted lines exist in the Chapter on "Superconductivity" in (Kachhava, 1990, p. 205).

" Then the differential Gibbs free energy is

$dG = - SdT + VdP - \frac{MdH}{\mu_0}$ ... (10.25)

At constant and , the free energy differences, because of the presence of a magnetic field, is found by integration. Thus

$$\int_{T,0}^{T,H} dG = -\int_{0}^{H} \frac{M}{\mu_0}\,dH$$

$$\mathrm{G(T,H)} - \mathrm{G(T,0)} = -\int_0^H \frac{M}{\mu_0}\,dH \text{ "}$$

(iv) The following quoted lines have been offered by the author in (Paul, 2019, p. 286).

"In deriving the second equation of motion inspired by calculus, it started with $v = \frac{ds}{dt}$, followed by $ds = vdt$, which can be expressed as $ds = (v_0 + at)\,dt$. The latter follows the integration of velocity to find position $\int_{s_0}^{s} ds = \int_0^t (v_0 + at)dt$."

(v) The following quoted lines have been incorporated by the authors in (Zhang & Li, 2018, p. 365).

"For reversible expansion or compression

$P_E = p \pm dp$, $\delta \mathrm{W} = - P_E\,dV$

For this process, the integral is:

$\mathrm{W} = -\int_{V_1}^{V_2} P_E\,dV = \cdots = \cdots$"

Examination of the information gathered from traditiodynal literature

It will now be interesting to consider the process of integration in the relation, $\int \frac{dy}{4\,-\,y2} = \int \frac{dx}{x}$ of the procedure of solution of Example 2 in case (i) of the previous section. In this case, we may think of the left hand side of this relation as, $\int \frac{dy}{4\,-\,y2} = \int \mathrm{d}\varphi(\mathrm{y})$, say. Similarly the right hand side of the aforesaid relation may be thought of as, $\int \frac{dx}{x} = \int d\psi(x)$, say. Thus the relation, $\int \frac{dy}{4\,-\,y2} = \int \frac{dx}{x}$ implies that $\int d\varphi(y) = \int d\psi(x)$. Now, it is easy to see that in the relation, $\int d\varphi(y) = \int d\psi(x)$, the left hand side is integrated with respect to the variable $\varphi(y)$ and the right hand side is integrated with respect to the variable $\psi(x)$. Since in general, the variables $\varphi(y)$ and $\psi(x)$ are not identical, such a process of integration with respect to one variable on one side and that with respect to another variable on the other side must have to be treated as an ambiguous one.

Thus it appears that the procedure of getting the last equation in the solution of Example 2 considered in case (i) of the previous section does not have any rationality with the usual notation for the process of integration.

Similar arguments could be made for the presence of ambiguity in each pat of the theoretical discussions considered in the previous section.

Discussions regarding the unambiguous procedural steps to be followed

In this section, the unambiguous procedural steps to be followed have been detailed with reference to each part of the theoretical discussions and the example problem considered above under the section 'Information gathered from traditional literature search regarding typical process of integration'.

In case (i), the equation under consideration is: $\frac{xdy}{dx} + y^2 = 4$

This equation may be written as

$$\frac{1}{4 - y2}\frac{dy}{dx} = \frac{1}{x}$$

or, $\frac{1}{4}\left\{\frac{1}{2 - y} + \frac{1}{2 + y}\right\}\frac{dy}{dx} = \frac{1}{x}$

or, $\frac{1}{4}\frac{d}{dx}\{log_e\ (2 + y)\} - \frac{1}{4}\frac{d}{dx}\{log_e\ (2 - y)\} = \frac{1}{x}$

Integrating both sides with respect to the variable x, we get,

$$\int\left[\frac{1}{4}\frac{d}{dx}\{log_e\ (2 + y)\}\right]dx - \int\left[\frac{1}{4}\frac{d}{dx}\{log_e\ (2 - y)\}\right]dx = \int\frac{1}{x}\ dx$$

or, $\frac{1}{4}\ log_e\ \left|\frac{2 + y}{2 - y}\right| = log_e\ |x| + log_e\ c$

In case (ii), the equation under consideration is:

$$ds = (u + ft)\,dt$$

This equation may be written as: $\frac{ds}{dt} = u + ft$

Integrating both sides with respect to the variable t, we then obtain

$$\int\left(\frac{ds}{dt}\right)dt = \int(u + ft)dt$$

$$\text{or, } s = ut + \frac{1}{2}ft^2 + C_2$$

In case (iii), the equation under consideration is:

$$dG = -\ SdT + VdP - \frac{MdH}{\mu_0}$$

At constant T and P, this equation becomes

$$\frac{dG}{dH} = -\frac{M}{\mu_0}$$

Integrating both sides with respect to the variable within the specified limits we get,

$$\int_{T,0}^{T,H}\frac{dG}{dH}\ dH = -\int_0^H\frac{M}{\mu_0}\ dH$$

$$\text{or,}\quad G(T,H) - G(T,0) = -\int_0^H\frac{M}{\mu_0}\ dH$$

In case (iv), the equation under consideration is: $ds = (v_0 + at)\,dt$

This equation can be re-written as: $\frac{ds}{dt} = (v_0 + at)$

Integrating both sides with respect to the variable "t" within proper limits, we then get,

$$\int_{s_0}^{s}\frac{ds}{dt}\ dt = \int_0^t(v_0 + at)dt$$

In case (v), the equation under consideration is: $\delta W = -P_E\, dV$

This equation can be integrated with respect to the variable V within proper limits to get,

$\int_0^W \frac{dW}{dV}\, dV = -\int_{V_1}^{V_2} P_E\, dV$; or, $W = -\int_{V_1}^{V_2} P_E\, dV$

It can be easily seen from above that there exists no ambiguity in the above procedure of performing the integration process for each of the cases from (i) to (v) since such a procedure does not violate the fundamental concept behind the notation of the operation of integration in Integral calculus.

Conclusion

In order to enhance deepening of thought and understanding regarding Integral calculus as well as to bring precision and sophistication in the relevant field of study, the fundamental concept in regard to the notation of the operation of integration has been revisited in this paper. Information gathered from the long-running literature on the use of the process of integration in various theoretical discussions and procedure of solution of example problem has been incorporated first and subsequently examined. The study reveals the discovery of ambiguity in the procedural steps of the integration process adopted for each part of the theoretical discussions and solution of the example problem considered, particularly because of the fact that each such procedure overlooks the proper use of the standard notation for the integration operation in which both the symbols '∫' and '' will have to appear at a time while performing an integration operation and the integrand is to be placed in between the said two symbols '∫' and ''. In order to get rid of such ambiguous procedure inherent in the traditional approaches (Dasgupta, 1997; De, 1998; Kachhava, 1990; Paul, 2019; Zhang & Li, 2018), the unambiguous procedural steps to be followed for each of those cases considered have been finally offered..

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Author biography

Pramode Ranjan Bhattacharjee was born at Agartala, India on 14 February 1952. He received the B.Sc. (Honours) and the M.Sc. degrees in Physics from Calcutta University, India in 1972 and 1974, respectively. In 1989, he received the Ph.D. degree from Jadavpur University, Calcutta, India for his works in the field of Fault detection of digital circuits. He joined as a lecturer in the Department of Physics, M.B.B.College, Agartala, Tripura, India on 11 July 1977 and subsequently rendered his services as Reader, Head of Department of Physics in the said college. At one time he was the Vice Principal (Academic) of M.B.B.College. He was the Principal of Kabi Nazrul Mahavidyalaya, Sonamura, Tripura, India during the period from 24 February 2010 to 29 February 2012. At present he is a Retired Principal of that college. He published several papers in India and abroad. His current research areas include basic Physics/Mathematics/ Engineering, and Physics/Mathematics/Engineering education. He is a Life member of IAPT and IJRULA. Once he was also an annual member of OSA for one year. He has been awarded with the International "Research Ratna Award" in 2019, awarded by RULA AWARDS and he won the award title "Best Researcher in Optical Physics". One of his articles published in IETE Journal of Education in 2018 has been selected as the best paper among those published in the said journal in 2018 and for that reason, he has been awarded with the "Students' Journal award" in 2019.