Centralized Multi-Sensor Robust Recursive Least-Squares Wiener Estimators in Linear Discrete-Time Stochastic Systems with Uncertain Parameters

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Abstract

Among the multi-sensor information fusion estimation problems, this paper proposes the centralized robust recursive least-squares (RLS) Wiener filter and fixed-point smoother for estimating the signal and the state in linear wide-sense stationary stochastic systems with the uncertain parameters in the system and observation matrices. Previously, the robust RLS Wiener filter and fixed-point smoother are proposed by the author in the case of the single-sensor observation for linear discrete-time stochastic systems with uncertain parameters. This paper extends the robust RLS Wiener estimators in the case of the single-sensor observation to the centralized multi-sensor robust RLS Wiener estimators. The signal is observed at each station as degraded by the uncertain parameters in the observation matrix. The centralized multi-sensor robust RLS Wiener filter and fixed-point smoother, proposed in this paper, have the advantage of not using information such as probabilities about the uncertain parameters in the system and observation matrices. Related to the centralized multi-sensor robust RLS Wiener filter, the recursive algorithm for the filtering error variance function of the state is proposed.

The estimation accuracies of the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother are superior to the centralized multi-sensor RLS Wiener filter and fixed-point smoother, respectively.

Keywords: Centralized robust RLS Wiener estimators; multi-sensor information fusion; base station; autoregressive model; uncertain stochastic systems

1. Introduction

Chen & Zhang (2011) design the robust Kalman filter in uncertain stochastic systems with time-invariant state delayed; bounded random observation delays and missing measurements in the case of the single-sensor observation. The redundant and complementary information, gained by the multi-sensor information fusion system, rather than using individual sensory data independently, enables us to get more accurate and less uncertain information (Zhang & Wei, 2014). A scalar weighting information fusion optimal Kalman filter has higher precision than each local filter in the case of the colored observation noise (Sun & Deng, 2005). Using the Kalman filtering technique, Sun (2004) proposes the multi-sensor optimal information fusion Kalman filter (IFKF). In the IFKF, the estimate of the state is calculated at the base station as the weighted sum of the estimates of the states at the local stations. Zhang, Qi & Deng (2014) devise the two-level robust measurement fusion Kalman filter over clustering sensor network systems with unknown noise variances. Zhang et al. (2016) propose the centralized fusion steady-state robust Kalman filter with the upper bound of noise variances. The multi-sensor information fusion estimators are proposed for linear stochastic uncertain systems, with packet losses (Ma & Sun, 2015) or with delayed observations (Qi, Zhang & Deng, 2014b). The multi-sensor information fusion robust estimators in linear discrete-time stochastic systems have been studied extensively for the systems with uncertain parameters (Luo, Zyu, Luo, Zhou, Song & Wang, 2008; Qi et al., 2014, 2014a). Chen, Yu, Zhang & Liu (2011) and Chen et al. (2013) propose the multi-sensor information fusion robust Kalman filter in the stochastic systems with parameter uncertainties, randomly delayed measurements and sensor failures.

By the way, the robust recursive least-squares (RLS) Wiener filter, fixed-point smoother (Nakamori, 2019a, 2019b), fixed-lag smoother (Nakamori, 2019c) and fixed-interval smoother (Nakamori, 2019d) are present. Nakamori designs these robust estimators in the case of the single-sensor observation and have not taken into account the multi-sensor information fusion estimation problems. From this, Theorem 1 presents the centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms for the signal and the state in linear discrete-time wide-sense stationary stochastic systems with the uncertain parameters in the system and observation matrices. The proposed centralized multi-sensor robust RLS Wiener filter and fixed-point smoother have the advantage of not using information such as probabilities about the uncertain parameters in the system and observation matrices. Theorem 2 presents the centralized multi-sensor RLS Wiener filtering and fixed-point smoothing algorithms for the signal and the state with the degraded observations. The centralized multi-sensor RLS Wiener filter and fixed-point smoother are obtainable as a natural extension of the RLS Wiener estimators (Nakamori, 1995).

A numerical example shows the estimation characteristics of the proposed centralized multi-sensor robust RLS Wiener filter and fixed-point smoother in comparison with the centralized multi-sensor RLS Wiener estimators. The estimation accuracies of the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother are superior to the centralized multi-sensor RLS Wiener filter and fixed-point smoother, respectively.

Section 2 introduces the centralized multi-sensor information fusion robust estimation problem in wide-sense stationary stochastic systems. In Section 3, Theorem 1 proposes the centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms. Theorem 2 presents the centralized multi-sensor RLS Wiener filtering and fixed-point smoothing algorithms. Related to the centralized multi-sensor robust RLS Wiener filter, Section 4 proposes the recursive algorithm for the filtering error variance function of the state, and shows the existence of the filtering estimate of the state. Section 5 demonstrates a numerical simulation example.

2. Degraded signals in linear multi-sensor wide-sense stationary stochastic systems

In linear discrete-time wide-sense stationary stochastic systems, at the local stations, let the multi-sensor signals $z_i(k)$, $i = 1,2,\cdots m$, be observed with additional observation noises $v_i(k)$ as follows when the state equation for $x(k)$ is given in (1).

$$
\begin{gathered}
y_i(k) = z_i(k) + v_i(k), z_i(k) = H_i x(k), i = 1,2,\cdots m,\\
y(k) = z(k) + v(k),\\
y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_m(k) \end{bmatrix}, z(k) = Hx(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \\ \vdots \\ z_m(k) \end{bmatrix}, H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \end{bmatrix}, v(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ \vdots \\ v_m(k) \end{bmatrix},\\
E[v_i(k)v_i^T(s)] = R_i\delta_K(k-s), E\left[v_i(k)v_j^T(s)\right] = 0, i \neq j, i,j = 1,2,\cdots m,\\
E[v(k)v^T(s)] = R\delta_K(k-s), R = \begin{bmatrix} R_1 & 0 & \cdots & 0 & 0 \\ 0 & R_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & R_{m-1} & 0 \\ 0 & 0 & \cdots & 0 & R_m \end{bmatrix},\\
x(k+1) = \Phi x(k) + \Gamma w(k), E[w(k)w^T(s)] = Q\delta_K(k-s)
\end{gathered}
\tag{1}
$$

Here, $z(k)$: $m \cdot M \times 1$ signal vector with components of m multi-sensor signals $z_i(k)$, $i = 1,2,\cdots m$; H_i: $M\times$ n observation matrix; $x(k)$: $n \times 1$ state vector to be estimated; $v_i(k)$: zero-mean white observation noise with variance R_i; Φ: state transition matrix; $w(k)$: white-noise input with variance Q; Γ: $n \times l$ input matrix. The notations $y(k), z(k)$ and $v(k)$ represent the stacked vectors of the $y_i(k), z_i(k)$ and $v_i(k)$ vectors, $i = 1,2,\cdots m$, respectively. The auto-covariance function of $v(k)$ is given in (1). Let the processes of the signals $z_i(k)$ and the observation noises $v_i(k)$ be independent mutually. Now, let the degraded multi-sensor observations $\breve{y}_i(k)$, $\mathrm{i} = 1,2,\ldots,\mathrm{m}$, be generated by the state-space model with the uncertain quantities $\Delta\Phi(k)$ in the system matrix and $\Delta\mathrm{H}_i(\mathrm{k})$ in the observation matrices. Let $\breve{y}_i(k)$, $i = 1,2,\cdots m$, be given as the sum of the degraded signal $\breve{z}_i(k)$ and the white observation noise $v_i(k)$ at the $\mathrm{i}-\mathrm{th}$ sensor.

$$
\begin{aligned}
&\breve{y}_i(k) = \breve{z}_i(k) + v_i(k), \breve{z}_i(k) = \breve{H}_i(k)\breve{x}(k),\\
&\breve{x}(k+1) = \breve{\Phi}(k)\breve{x}(k) + \Gamma w(k),\\
&\breve{\Phi}(k) = \Phi + \Delta\Phi(k), \breve{H}_i(k) = H_i + \Delta\mathrm{H}_i(k), \mathrm{i} = 1,\ldots,\mathrm{m}
\end{aligned}
\tag{2}
$$

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Let the notations $\breve{y}(k)$ and $\breve{z}(k)$ denote the stacked vectors of the $\breve{y}_i(k)$ and $\breve{z}_i(k)$ vectors, $i = 1,2,\cdots m$, respectively. Then the observation equations in (2) are expressed with the stacked vectors as follows.

$$\breve{y}(k) = \breve{z}(k) + v(k),$$
$$\breve{y}(k) = \begin{bmatrix} \breve{y}_1(k) \\ \breve{y}_2(k) \\ \vdots \\ \breve{y}_m(k) \end{bmatrix}, \breve{z}(k) = \begin{bmatrix} \breve{z}_1(k) \\ \breve{z}_2(k) \\ \vdots \\ \breve{z}_m(k) \end{bmatrix}. \tag{3}$$

Let the process of the degraded multi-sensor signal $\breve{z}(k)$ be fitted to the multivariate autoregressive (AR) model of the finite order N as follows.

$$\breve{z}(k) = -\overleftrightarrow{a}_1\breve{z}(k-1) - \overleftrightarrow{a}_2\breve{z}(k-2) \cdots - \overleftrightarrow{a}_N\breve{z}(k-N) + \overleftrightarrow{e}(k),$$
$$E[\overleftrightarrow{e}(k)\overleftrightarrow{e}^T(s)] = \overleftrightarrow{Q}\delta_K(k-s) \tag{4}$$

In (Nakamori, 2019a), the degraded signal is observed at a single station of m= 1, where the degraded signal is expressed in terms of the AR model of the finite order. In the case of the multi-sensor observations for $m \geq 2$, we introduce the multi-sensor state $\overleftrightarrow{x}(k)$, with components $\breve{z}_1(k)$, $\breve{z}_2(k)$, $\breve{z}_3(k)$, ..., $\breve{z}_m(k)$, $\breve{z}_1(k+1)$, $\breve{z}_2(k+1)$, ..., $\breve{z}_m(k+1)$, $\cdots$,$\breve{z}_1(k+N-2)$, $\breve{z}_2(k+N-2)$, $\cdots$, $\breve{z}_m(k+N-2)$, $\breve{z}_1(k+N-1)$, $\breve{z}_2(k+N-1)$, $\cdots$, $\breve{z}_m(k+N-1)$. By introducing the observation matrix $\overleftrightarrow{H}$ and the state $\overleftrightarrow{x}(k)$, the degraded multi-sensor signal $\breve{z}(k)$ is expressed as

$$\breve{z}(k) = \overleftrightarrow{H}\overleftrightarrow{x}(k), \overleftrightarrow{H} = [I_{M\cdot m\times M\cdot m} \quad 0 \quad \cdots \quad 0 \quad 0],$$
$$\overleftrightarrow{x}(k) = \begin{bmatrix} \breve{z}(k) \\ \breve{z}(k+1) \\ \vdots \\ \breve{z}(k+N-2) \\ \breve{z}(k+N-1) \end{bmatrix} = \begin{bmatrix} \breve{z}_1(k) \\ \breve{z}_2(k) \\ \vdots \\ \breve{z}_m(k) \\ \breve{z}_1(k+1) \\ \breve{z}_2(k+1) \\ \vdots \\ \breve{z}_m(k+1) \\ \vdots \\ \breve{z}_1(k+N-2) \\ \breve{z}_2(k+N-2) \\ \vdots \\ \breve{z}_m(k+N-2) \\ \breve{z}_1(k+N-1) \\ \breve{z}_2(k+N-1) \\ \vdots \\ \breve{z}_m(k+N-1) \end{bmatrix}. \tag{5}$$

From (4) and (5), the state equation for $\overleftrightarrow{x}(k)$ is given by

$$\overleftrightarrow{x}(k+1) = \overleftrightarrow{\Phi}\overleftrightarrow{x}(k) + \overleftrightarrow{\Gamma}\overleftrightarrow{w}(k),$$
$$\overleftrightarrow{\Phi} = \begin{bmatrix} 0 & I_{M\cdot m\times M\cdot m} & 0 & \cdots & 0 \\ 0 & 0 & I_{M\cdot m\times M\cdot m} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_{M\cdot m\times M\cdot m} \\ -\overleftrightarrow{a}_N & -\overleftrightarrow{a}_{M-1} & -\overleftrightarrow{a}_{M-2} & \cdots & -\overleftrightarrow{a}_1 \end{bmatrix}, \overleftrightarrow{\Gamma} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I_{M\cdot m\times M\cdot m} \end{bmatrix}, \tag{6}$$
$$E[\overleftrightarrow{w}(k)\overleftrightarrow{w}^T(s)] = \overleftrightarrow{Q}\delta_K(k-s), \overleftrightarrow{w}(k) = \overleftrightarrow{e}(k+N),$$

using the system matrix $\overleftrightarrow{\Phi}$ in the controllable canonical form. The AR parameters $\overleftrightarrow{a}_i$, $1 \le i \le N$, are calculated, by the Yule-Walker equations (Nakamori, 2019b), using the auto-covariance function of the degraded multi-sensor signal $\breve{z}(k)$, $\breve{K}(k,s) = E[\breve{z}(k)\breve{z}^T(s)] = \breve{K}(i)$, $i = k - s$, $0 \le i \le N$.

$$\overleftrightarrow{K}(k,k)\begin{bmatrix} \overleftrightarrow{a}_1^T \\ \overleftrightarrow{a}_2^T \\ \vdots \\ \overleftrightarrow{a}_{N-1}^T \\ \overleftrightarrow{a}_N^T \end{bmatrix} = -\begin{bmatrix} \breve{K}^T(1) \\ \breve{K}^T(2) \\ \vdots \\ \breve{K}^T(N-1) \\ \breve{K}^T(N) \end{bmatrix} \quad (7)$$

Here, the auto-variance function $\overleftrightarrow{K}(k,k)$ of the multi-sensor state $\overleftrightarrow{x}(k)$ is expressed as follows

$$\overleftrightarrow{K}(k,k) = E\left[\overleftrightarrow{x}(k)\overleftrightarrow{x}^T(k)\right]$$
$$= \begin{bmatrix} \breve{K}(0) & \breve{K}^T(1) & \cdots \breve{K}^T(N-2) & \breve{K}^T(N-1) \\ \breve{K}(1) & \breve{K}(0) & \cdots \breve{K}^T(N-3) & \breve{K}^T(N-2) \\ \vdots & \vdots & \ddots \quad \vdots & \vdots \\ \breve{K}(N-2) & \breve{K}(N-3) & \cdots \quad \breve{K}(0) & \breve{K}^T(1) \\ \breve{K}(N-1) & \breve{K}(N-2) & \cdots \quad \breve{K}(1) & \breve{K}(0) \end{bmatrix}. \quad (8)$$

Also, the equation for the cross-covariance function $K_{x\overleftrightarrow{x}}(k,s) = E[x(k)\overleftrightarrow{x}^T(s)]$ of the state $x(k)$ with the state $\overleftrightarrow{x}(s)$ is

$$K_{x\overleftrightarrow{x}}(k,s) = \alpha(k)\beta^T(s), 0 \le s \le k,$$
$$\alpha(k) = \Phi^k, \beta^T(s) = \Phi^{-s}K_{x\overleftrightarrow{x}}(s,s). \quad (9)$$

3. Centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms in linear stochastic systems with uncertain parameters

Under the centralized multi-sensor robust estimation problem in Section 2, Theorem 1 presents the centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms for estimating the signal $z(k)$ and the state $x(k)$ in linear wide-sense stationary stochastic systems with the uncertain parameters in the system and observation matrices.

Theorem 1 Let the state-space model for the state $x(k)$ be given by (1). Let the state-space model with the uncertain quantities $\Delta\Phi(k)$ and $\Delta H_i(k)$, $i = 1, \dots, m$, be given by (2). Let the process of the degraded multi-sensor signal $\breve{z}(k)$ be fitted to the AR model of the order N. Let the variance $\overleftrightarrow{K}(k,k)$ of the multi-sensor state $\overleftrightarrow{x}(k)$ and the cross-variance $K_{x\overleftrightarrow{x}}(k,k)$ of the state $x(k)$ with the multi-sensor state $\overleftrightarrow{x}(k)$ be given by (8) and (9), respectively. Let the variance of the multi-sensor white observation noise $v(k)$ be R. Then the centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms for the signal $z(k)$ and the state $x(k)$ consist of (10)-(20) in linear wide-sense stationary stochastic systems with the uncertain parameters in the system and observation matrices.

Fixed-point smoothing estimate of the signal $z(k)$ at the fixed point k: $\hat{z}(k,L)$

$$\hat{z}(k,L) = H\hat{x}(k,L) \quad (10)$$

Fixed-point smoothing estimate of the state $x(k)$ at the fixed point k: $\hat{x}(k,L)$

$$\hat{x}(k,L) = \hat{x}(k,L-1) + h(k,L,L)(\breve{y}(L) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(L-1,L-1)),$$
$$\hat{x}(k,L)|_{L=k} = \hat{x}(k,k) \quad (11)$$

Smoother gain for $\hat{x}(k,L)$ in (11): $h(k,L,L)$

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$$h(k,L,L) = [K_{x\overleftrightarrow{x}}(k,k)(\overleftrightarrow{\Phi}^T)^{L-k}\overleftrightarrow{H}^T - q(k,L-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T] \times \{R + \overleftrightarrow{H}[\overleftrightarrow{K}(L,L) - \overleftrightarrow{\Phi}S_0(L-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T\}^{-1} \quad (12)$$

$$q(k,L) = q(k,L-1) + h(k,L,L)\overleftrightarrow{H}[\overleftrightarrow{K}(L,L) - \overleftrightarrow{\Phi}S_0(L-1)\overleftrightarrow{\Phi}^T], \quad q(k,k) = S_0(k) \quad (13)$$

Filtering estimate of the signal $z(k)$: $\hat{z}(k,k)$

$$\hat{z}(k,k) = H\hat{x}(k,k) \quad (14)$$

Filtering estimate of the state $x(k)$: $\hat{x}(k,k)$

$$\hat{x}(k,k) = \Phi\hat{x}(k-1,k-1) + G(k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)), \quad \hat{x}(0,0) = 0 \quad (15)$$

Filter gain for $\hat{x}(k,k)$ in (15): $G(k)$

$$G(k) = [K_{x\breve{z}}(k,k) - \Phi S(k-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T] \times \{R + \overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T\}^{-1}, \quad K_{x\breve{z}}(k,k) = K_{x\overleftrightarrow{x}}(k,k)\overleftrightarrow{H}^T \quad (16)$$

Filtering estimate of $\overleftrightarrow{x}(k)$: $\hat{\overleftrightarrow{x}}(k,k)$

$$\hat{\overleftrightarrow{x}}(k,k) = \overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1) + g(k)(\breve{y}(k) - \overleftrightarrow{H}\overleftrightarrow{\Phi}\hat{\overleftrightarrow{x}}(k-1,k-1)), \quad \hat{\overleftrightarrow{x}}(0,0) = 0 \quad (17)$$

Filter gain for $\hat{\overleftrightarrow{x}}(k,k)$ in (17): $g(k)$

$$g(k) = [\overleftrightarrow{K}(k,k)\overleftrightarrow{H}^T - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T\overleftrightarrow{H}^T] \times \{R + \overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T\}^{-1} \quad (18)$$

Auto-variance function of $\hat{\overleftrightarrow{x}}(k,k)$: $S_0(k) = E[\hat{\overleftrightarrow{x}}(k,k)\hat{\overleftrightarrow{x}}^T(k,k)]$

$$S_0(k) = \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T + g(k)\overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T], \quad S_0(0) = 0 \quad (19)$$

Cross-variance function of $\hat{x}(k,k)$ with $\hat{\overleftrightarrow{x}}(k,k)$: $S(k) = E[\hat{x}(k,k)\hat{\overleftrightarrow{x}}^T(k,k)]$

$$S(k) = \Phi S(k-1)\overleftrightarrow{\Phi}^T + G(k)\overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T], \quad S(0) = 0 \quad (20)$$

Theorem 1 is obtained by extending the robust RLS Wiener estimators (Nakamori, 2019a, 2019b) for the case of the single-sensor observation to the centralized multi-sensor robust RLS Wiener estimators in linear discrete-time stochastic systems with the uncertain parameters.

For the stability of the centralized multi-sensor robust RLS Wiener estimators, from (12), (15), (16), (17) and (18), the following conditions are required. (1) The matrix $R + \overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T$ is positive definite. (2) The system matrix Φ is stable. (3) The matrix $\overleftrightarrow{\Phi} - g(k)\overleftrightarrow{H}\overleftrightarrow{\Phi}$ is stable. Namely, (2) and (3) mean that all the eigenvalues of the matrices Φ and $\overleftrightarrow{\Phi} - g(k)\overleftrightarrow{H}\overleftrightarrow{\Phi}$ lie inside the unit circle, respectively.

Theorem 2 proposes the centralized multi-sensor RLS Wiener filtering and fixed-point smoothing algorithms.

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Theorem 2 Let the state-space model for the state $x(k)$ be given by of (1). Then the centralized multi-sensor RLS Wiener filtering and fixed-point smoothing algorithms for the signal $z(k)$ and the state $x(k)$ consist of (21)-(28). The centralized multi-sensor RLS Wiener estimators use the degraded multi-sensor observed value $\breve{y}(k)$ instead of the observation $y(k)$ in (1). The centralized multi-sensor RLS Wiener filtering and fixed-point smoothing algorithms use the system matrix Φ, the observation matrix H, the auto-variance function of $x(k)$, $K_x(k,k) = E[x(k)x^T(k)]$, and the degraded multi-sensor observed value $\breve{y}(k)$ in linear discrete-time wide-sense stationary stochastic systems.

Fixed-point smoothing estimate of the signal $z(k)$: $\hat{z}(k,L)$

$$\hat{z}(k,L) = H\hat{x}(k,L) \tag{21}$$

Fixed-point smoothing estimate of the state $x(k)$: $\hat{x}(k,L)$

$$\begin{aligned} &\hat{x}(k,L) = \hat{x}(k,L-1) + h(k,L,L)(\breve{y}(L) - H\Phi\hat{x}(L-1,L-1)), \\ &\hat{x}(k,L)|_{L=k} = \hat{x}(k,k) \end{aligned} \tag{22}$$

Smoother gain for $\hat{x}(k,L)$ in (22): $h(k,L,L)$

$$\begin{aligned} &h(k,L,L) = [K_x(k,k)(\Phi^T)^{L-k}H^T - q(k,L-1)\Phi^T H^T] \\ &\times \{R + H[K_x(L,L) - \Phi S_x(L-1)\Phi^T]H^T\}^{-1} \end{aligned} \tag{23}$$

$$\begin{aligned} &q(k,L) = q(k,L-1)\Phi^T + h(k,L,L)H[K_x(L,L) - \Phi S_x(L-1)\Phi^T], \\ &q(k,k) = S_x(k) \end{aligned} \tag{24}$$

Filtering estimate of the signal $z(k)$: $\hat{z}(k,k)$

$$\hat{z}(k,k) = H\hat{x}(k,k) \tag{25}$$

Filtering estimate of the state $x(k)$: $\hat{x}(k,k)$

$$\begin{aligned} &\hat{x}(k,k) = \Phi\hat{x}(k-1,k-1) + G_x(k)(\breve{y}(k) - H\Phi\hat{x}(k-1,k-1)), \\ &\hat{x}(0,0) = 0 \end{aligned} \tag{26}$$

Filter gain for $\hat{x}(k,k)$ in (26): $G_x(k)$

$$\begin{aligned} &G_x(k) = [(K_x(k,k) - \Phi S_x(k-1)\Phi^T)H^T] \\ &\times \{R + H[K_x(k,k) - \Phi S_x(L-1)\Phi^T]H^T\}^{-1} \end{aligned} \tag{27}$$

Variance of filtering estimate $\hat{x}(k,k)$: $S_x(k)$

$$\begin{aligned} &S_x(k) = \Phi S_x(k-1)\Phi^T + G_x(k)H[K_x(k,k) - \Phi S_x(k-1)\Phi^T], \\ &S_x(0) = 0 \end{aligned} \tag{28}$$

Theorem 2 is obtained, in a straightforward manner, by an extension of the RLS Wiener estimators (Nakamori, 1995) to the case of the multi-sensor observations.

Section 4 presents the recursive algorithm for the filtering error variance function of the state $x(k)$ for the centralized multi-sensor robust RLS Wiener filtering algorithm, and the existence of the state is shown.

4. Filtering error variance function of state $x(k)$

This section, at first, proposes the recursive algorithm for the filtering error variance function $P_{\tilde{x}}(k,k)$ of the state $x(k)$ for the centralized multi-sensor robust RLS Wiener filtering algorithm. From (15) and (16), the variance $P_{\hat{x}}(k,k)$ of the filtering estimate $\hat{x}(k,k)$ is given by

$$\mathrm{E}[\hat{x}(k,k)\hat{x}^T(k,k)] = \Phi \mathrm{E}[\hat{x}(k-1,k-1)\hat{x}^T(k-1,k-1)]\Phi^T + G(k)[\breve{R} + \overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi}S_0(k-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T]G^T(k).$$

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That is, $P_{\hat{x}}(k,k)$ is updated by

$$P_{\hat{x}}(k,k) = \Phi P_{\hat{x}}(k-1,k-1)\Phi^T + G(k)[\breve{R} + \overleftrightarrow{H}[\overleftrightarrow{K}(k,k) - \overleftrightarrow{\Phi} S_0(k-1)\overleftrightarrow{\Phi}^T]\overleftrightarrow{H}^T]G^T(k),$$
$$P_{\hat{x}}(0,0) = 0. \quad (29)$$

The filtering error variance function $P_{\tilde{x}}(k,k)$ of $x(k)$ is given by

$$P_{\tilde{x}}(k,k) = K_x(k,k) - P_{\hat{x}}(k,k). \quad (30)$$

Here, $K_x(k,k)$ and $P_{\hat{x}}(k,k)$ represent the variance of the state $x(k)$ and the variance of the filtering estimate $\hat{x}(k,k)$, respectively. It is noted that the filtering error variance function $P_{\tilde{x}}(k,k)$ is calculated by (16), (18), (19), (20), (29) and (30) recursively.

From $K_x(k,k) \geq 0$ and $P_{\tilde{x}}(k,k) \geq 0$, the following relationship holds.

$$0 \leq P_{\hat{x}}(k,k) \leq K_x(k,k)$$

The variance of the filtering estimate of the state is lower bounded by the zero matrix and upper bounded by the variance of the state. This validates the existence of the filtering estimate $\hat{x}(k,k)$ of the state $x(k)$.

Section 5 shows a numerical simulation example of the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother in Theorem 1. We compare their estimation accuracies with those of the centralized multi-sensor RLS Wiener estimators of Theorem 2.

5. A numerical simulation example

Let the observation equations in the two-sensor information fusion network system of $\mathrm{m} = 2$ and the state equation for $x(k)$ be given by

$$y_i(k) = z_i(k) + v_i(k), z_i(k) = H_i x(k), i = 1,2,$$
$$y(k) = z(k) + v(k), z(k) = Hx(k), H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix},$$
$$H_1 = [1 \quad -0.1], H_2 = [0.1 \quad 1],$$
$$y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}, z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} = \begin{bmatrix} x_1(k) - 0.1x_2(k) \\ 0.1x_1(k) + x_2(k) \end{bmatrix}, v(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}, \quad (31)$$
$$x(k+1) = \Phi x(k) + \Gamma w(k), \Phi = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$E[v(k)v(s)] = R\delta_K(k-s), R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, R_1 = R_2,$$
$$E[w(k)w(s)] = Q\delta_K(k-s), Q = 0.5^2.$$

Let the observation equation for the degraded observed value $\breve{y}(k)$ be given by (32). The degraded observation $\breve{y}(k)$ consists of the two components $\breve{y}_1(k)$ and $\breve{y}_2(k)$, and the degraded signal $\breve{z}(k)$ consists of the two components $\breve{z}_1(k)$ and $\breve{z}_2(k)$ as follows.

$$\breve{y}(k) = \breve{z}(k) + v(k), \breve{z}(k) = \breve{H}(k)\breve{x}(k), \breve{y}(k) = \begin{bmatrix} \breve{y}_1(k) \\ \breve{y}_2(k) \end{bmatrix}, \breve{z}(k) = \begin{bmatrix} \breve{z}_1(k) \\ \breve{z}_2(k) \end{bmatrix},$$
$$\breve{H}(k) = \begin{bmatrix} \breve{H}_1(k) \\ \breve{H}_2(k) \end{bmatrix}, \breve{x}(k) = \begin{bmatrix} \breve{x}_1(k) \\ \breve{x}_2(k) \end{bmatrix} \quad (32)$$

Here, the state-space model contains the uncertain quantities $\Delta \mathrm{H}_i(k), i = 1,2$, and $\Delta\Phi(k)$ as follows.

$$\breve{y}_i(k) = \breve{z}_i(k) + v_i(k), \breve{z}_i(k) = \breve{H}_i(k)\breve{x}(k),$$
$$\breve{x}(k+1) = \breve{\Phi}(k)\breve{x}(k) + \Gamma w(k), \quad (33)$$
$$\breve{\Phi}(k) = \Phi + \Delta\Phi(k), \breve{H}_i(k) = H_i + \Delta \mathrm{H}_i(k), i = 1,2,$$

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$$\Delta\Phi(k) = \begin{bmatrix} 0 & 0 \\ 0.2\zeta_1(k) & 0.1\zeta_2(k) \end{bmatrix},$$
$$\Delta\mathrm{H}_1(k) = [0.1\zeta_3(k) \quad 0],\ \Delta\mathrm{H}_2(k) = [0.05\zeta_4(k) \quad 0]$$

It should be noted that the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 do not use the information related to the uncertain quantities. $\zeta_i(k)$, $\mathrm{i} = 1,2,\dots,4$, are mutually independent uniformly distributed random variables taking values in the range 0 to 1. The degraded multi-sensor signal $\breve{z}(k)$ is fitted to the multivariate AR model (4) of the order $\mathrm{N} = 5$ as an example. Thus, the multi-sensor state $\overleftrightarrow{x}(k)$ of (5) consists of 10 vector components.

By substituting H, Φ, $\overleftrightarrow{H}$, $\overleftrightarrow{\Phi}$, $\overleftrightarrow{K}(L,L)$, $K_{x\overleftrightarrow{x}}(k,k)$ and R into Theorem 1, the centralized robust RLS Wiener filtering and fixed-point smoothing estimates of the states $x_1(k)$ and $x_2(k)$ are calculated. Here, in the evaluations of $\overleftrightarrow{K}(L,L)$ and $K_{x\overleftrightarrow{x}}(k,k)$, $x(k)$ and $\overleftrightarrow{x}(k)$, $1 \le \mathrm{k} \le 350$, are used. The observed values are degraded by the uncertain parameters in the system and observation matrices. Fig. 1 illustrates the state $x_1(\mathrm{k})$, the filtering estimate $\hat{x}_1(k,k)$ and the fixed-point smoothing estimate $\hat{x}_1(k,k+5)$ vs. time k by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$. Fig. 2 illustrates the state $x_2(\mathrm{k})$, the filtering estimate $\hat{x}_2(\mathrm{k},\mathrm{k})$ and the fixed-point smoothing estimate $\hat{x}_2(k,k+5)$ vs. time k by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$. The centralized multi-sensor RLS Wiener estimators in Theorem 2 use the information Φ, H, and the auto-variance function of the state $x(k)$, $K_x(k,k)$. $K_x(k,k)$ equals $K_x(0)$ in wide-sense stationary stochastic systems. $K_x(k,k)$ is calculated by $K_x(k+1,k+1) = \Phi K_x(k,k)\Phi^T + \Gamma \mathrm{Q} \Gamma^T$, with the initial value $K_x(k,k) = 0_{2\times 2}$, iteratively until $K_x(k,k)$ attains its stationary value.

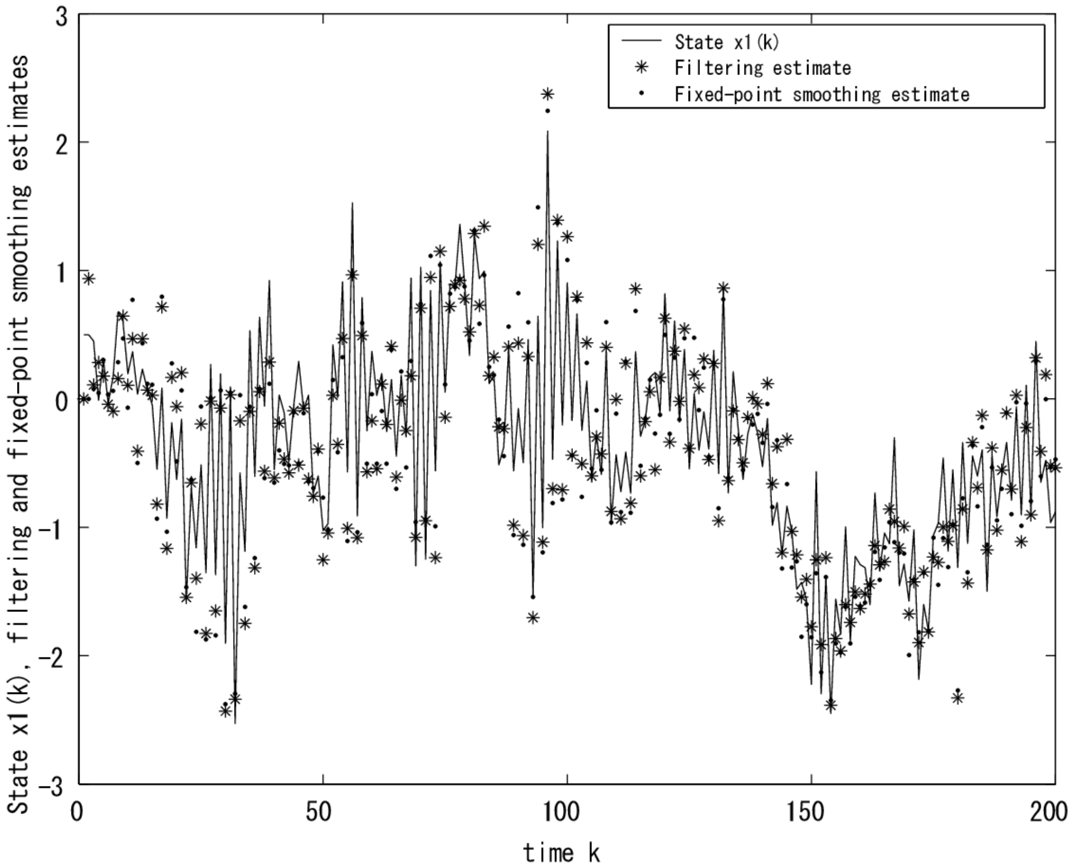

Fig. 1 State $x_1(k)$, the filtering estimate $\hat{\mathrm{x}}_1(\mathrm{k},\mathrm{k})$ and the fixed-point smoothing estimate $\hat{\mathrm{x}}_1(k,k+5)$ by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 vs. time k using the observations degraded by the uncertain parameters in the system and observation matrices for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$.

Fig. 3 illustrates the mean-square values (MSVs) of the filtering errors $x_1(k) - \hat{x}_1(k,k)$ and the fixed-point smoothing errors $x_1(\mathrm{k}) - \hat{x}_1(k,k+Lag)$, $1 \le \mathrm{k} \le 2000$, vs. Lag, $1 \le \mathrm{Lag} \le 10$, by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 and the centralized multi-sensor RLS Wiener estimators in Theorem 2, using the observations degraded by the uncertain parameters in the system and observation matrices, for the white Gaussian observation noises $\mathrm{N}(0,0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0,1.5^2)$. Fig. 4 illustrates the MSVs of the filtering errors $x_2(k) - \hat{x}_2(k,k)$ and the fixed-point smoothing errors $x_2(\mathrm{k}) - \hat{x}_2(k,k+Lag)$, $1 \le \mathrm{k} \le 2000$, vs. Lag, $1 \le \mathrm{Lag} \le 10$, by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 and the centralized multi-sensor RLS Wiener estimators in Theorem 2, using the observations degraded by the uncertain parameters in the system and observation matrices, for the white Gaussian observation noises $\mathrm{N}(0,0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0,1.5^2)$. From Fig. 3 and Fig. 4, the followings can be seen on the estimation characteristics of both $x_1(k)$ and $x_2(k)$.

(1) The MSVs of the fixed-point smoothing errors by the centralized multi-sensor robust fixed-point smoother are convergent for each observation noise. For the observation noises $\mathrm{N}(0,0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0,1.5^2)$, the MSVs of the fixed-point smoothing errors $x_i(\mathrm{k}) - \hat{x}_i(k,k+Lag), \mathrm{i} = 1,2$, decrease little by little as Lag increases for $1 \le Lag \le 3$.

(2) The estimation accuracies of the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother are superior to the centralized multi-sensor RLS Wiener filter and fixed-point smoother, respectively, for the observation noises $\mathrm{N}(0,0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0,1.5^2)$. In the centralized multi-sensor RLS Wiener fixed-point smother, as Lag increases, $2 \le Lag \le 10$, the MSV of the fixed-point smoothing errors increases for each observation noise.

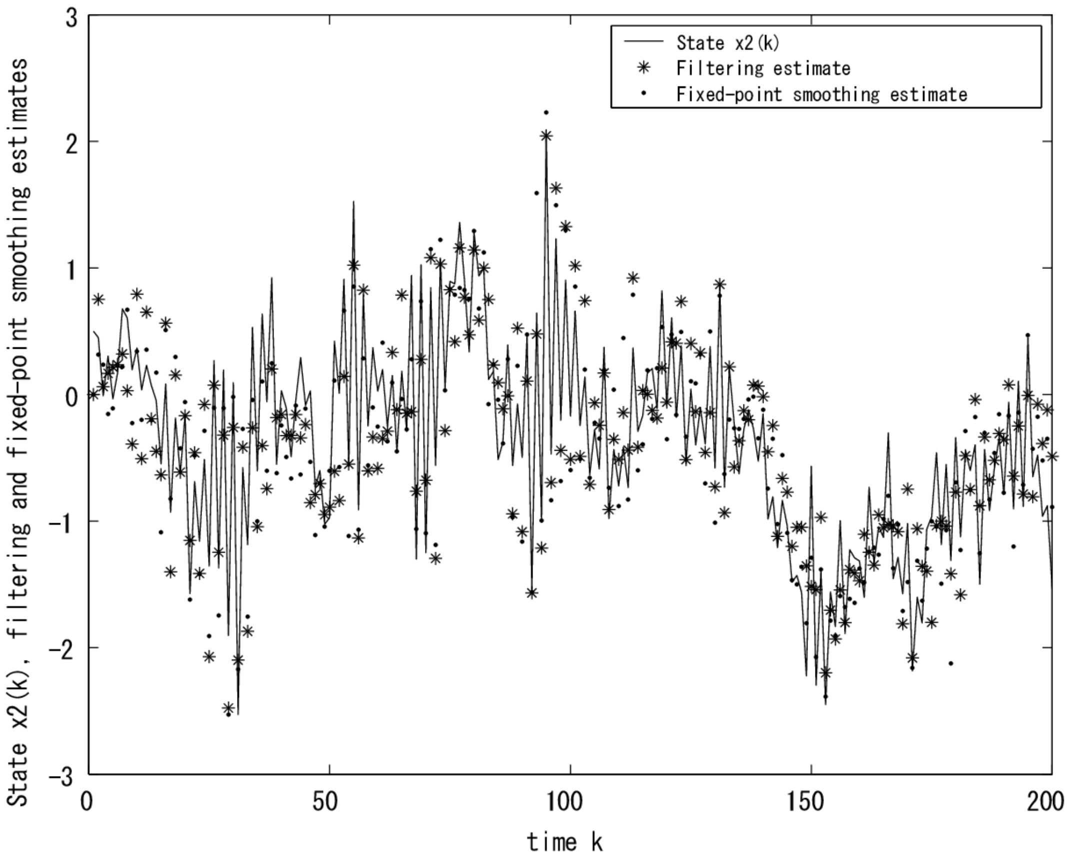

Fig. 2 State $x_2(k)$, the filtering estimate $\hat{x}_2(\mathrm{k},\mathrm{k})$ and the fixed-point smoothing estimate $\hat{x}_2(k,k+5)$ by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 vs. time k using the observations degraded by the uncertain parameters in the system and observation matrices for the white Gaussian observation noise $\mathrm{N}(0,0.5^2)$.

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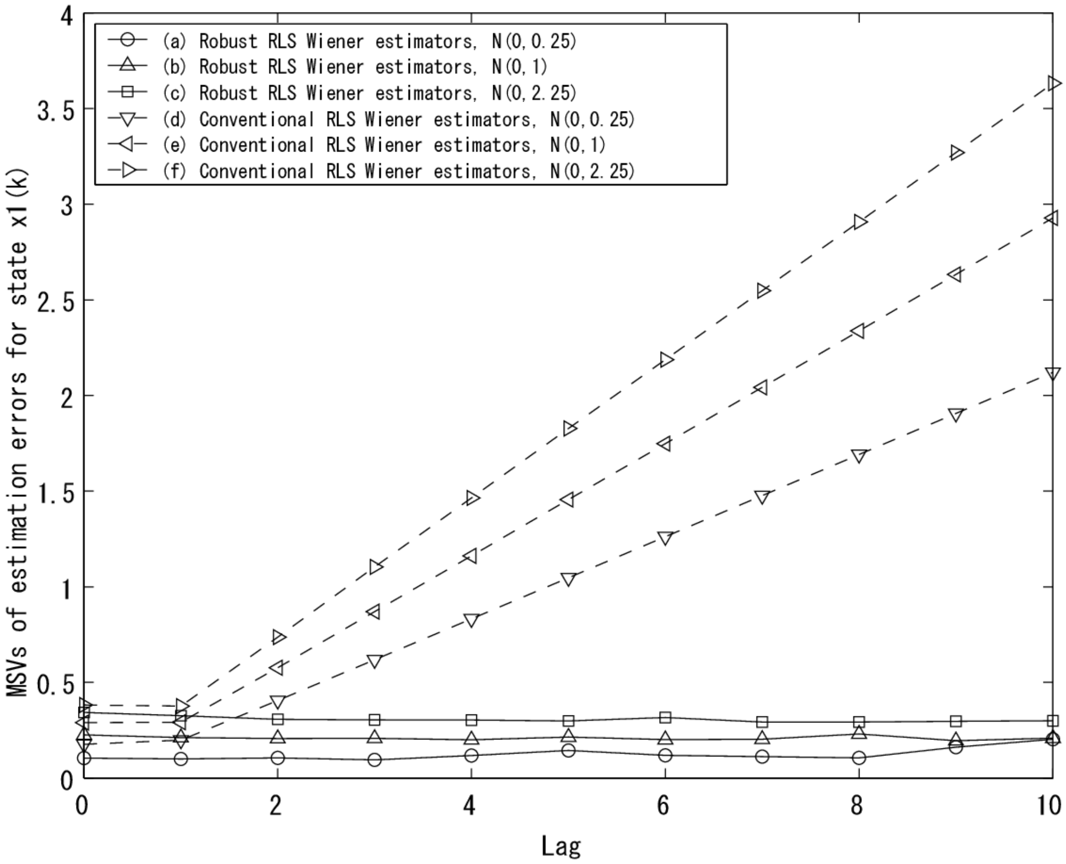

Fig. 3 Mean-square values of the filtering errors $x_1(k) - \hat{x}_1(k,k)$ and the fixed-point smoothing errors $x_1(\mathrm{k}) - \hat{x}_1(k, k + Lag)$, $1 \le \mathrm{Lag} \le 10$, $1 \le \mathrm{k} \le 2000$, vs. Lag by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 and the centralized multi-sensor RLS Wiener estimators in Theorem 2 using the observations degraded by the uncertain parameters in the system and observation matrices for the white observation noises $\mathrm{N}(0, 0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0, 1.5^2)$.

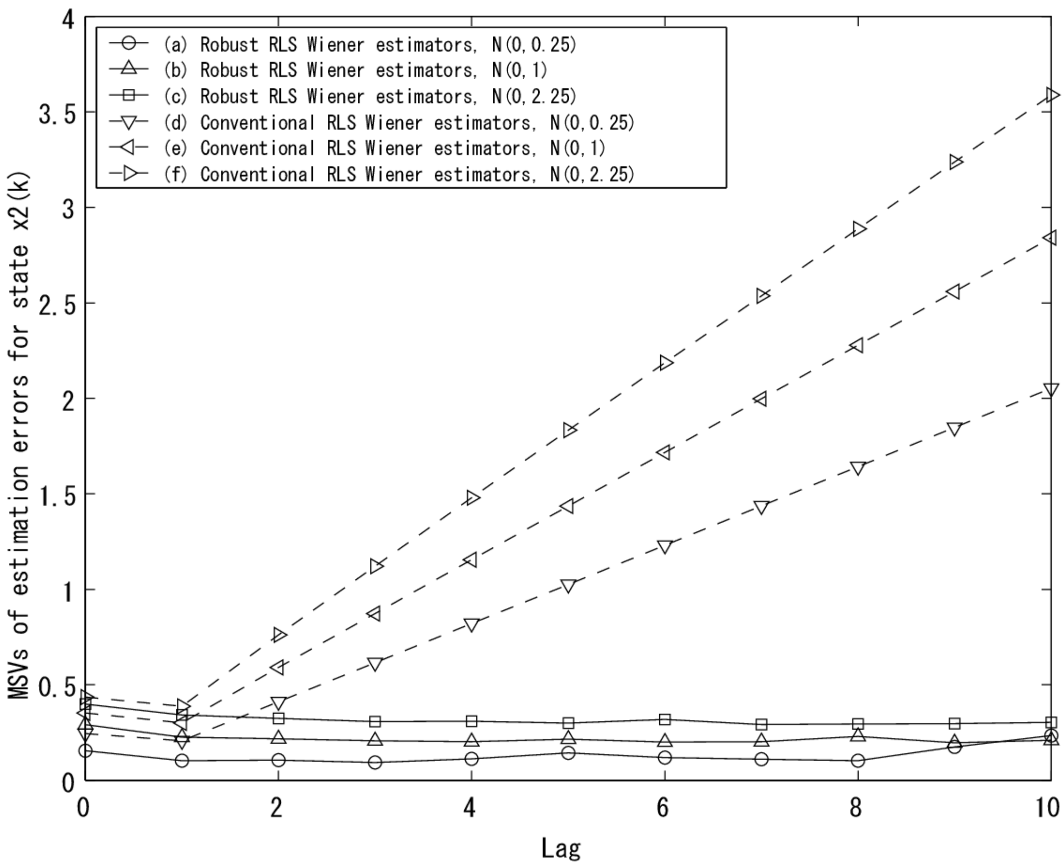

Fig. 4 Mean-square values of the filtering errors $x_2(k) - \hat{x}_2(k,k)$ and the fixed-point smoothing errors $x_2(\mathrm{k}) - \hat{x}_2(k, k + Lag)$, $1 \le \mathrm{Lag} \le 10$, $1 \le \mathrm{k} \le 2000$, vs. Lag by the centralized multi-sensor robust RLS Wiener estimators in Theorem 1 and the centralized multi-sensor RLS Wiener estimators in Theorem 2 using the observations degraded by the uncertain parameters in the system and observation matrices for the white observation noises $\mathrm{N}(0, 0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0, 1.5^2)$.

6. Conclusions

This paper has proposed, in Theorem 1, the centralized multi-sensor robust RLS Wiener filtering and fixed-point smoothing algorithms for the signal and the state in linear discrete-time wide-sense stationary stochastic systems with uncertain parameters. Theorem 2 has proposed the centralized RLS Wiener filtering and fixed-point smoothing algorithms for the signal and the state. Section 4 has proposed the recursive algorithm for the filtering error variance function of the state $x(k)$ for the centralized multi-sensor robust RLS Wiener filter, and has shown the existence of the state.

In the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother, the MSVs of the fixed-point smoothing errors for $x_1(\mathrm{k})$ and $x_2(\mathrm{k})$ are convergent for each observation noise. The MSVs of the fixed-point smoothing errors $x_i(\mathrm{k}) - \hat{x}_i(k, k + Lag)$, $\mathrm{i} = 1,2$, decrease little by little as Lag increases for $1 \le Lag \le 3$. For the estimations of both $x_1(k)$ and $x_2(k)$, the estimation accuracies of the centralized multi-sensor robust RLS Wiener filter and fixed-point smoother are superior to the centralized RLS Wiener filter and fixed-point smoother, respectively, for the observation noises $\mathrm{N}(0, 0.5^2)$, $\mathrm{N}(0,1)$ and $\mathrm{N}(0, 1.5^2)$. In the centralized RLS Wiener fixed-point smother, as Lag increases, $2 \le \mathrm{Lag} \le 10$, the MSVs of the fixed-point smoothing errors for $x_1(\mathrm{k})$ and $x_2(\mathrm{k})$ tend to be large for each observation noise.

References

Chen, B., Yu, L., Zhang, W.-A., & Liu, A. (2011). Distributed robust fusion Kalman filtering for uncertain stochastic systems with random delays and missing measurements. *2011 8th Asian Control Conference*

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(ASCC), Kaohsiung, Taiwan, IEEE Catalog Number: CFP11832-PRT, 1288–1293. https://ieeexplore.ieee.org/document/5899258

Chen, B., & Zhang, W.-A. (2011). Robust Kalman filtering for uncertain state delay systems with random observation delays and missing measurements. *IET Control Theory and Applications*, *5*(17), 1945–1954. https://digital-library.theiet.org/content/journals/10.1049/iet-cta.2010.0685

Chen, B., Yu, L., Zhang, W.-A., & Liu, A. (2013). Robust information fusion estimator for multiple delay-tolerant sensors with different failure rates. *IEEE Transactions on Circuits and Systems*, *60*(2), 401–414. https://ieeexplore.ieee.org/abstract/document/6338309, DOI: 10.1109/TCSI.2012.2215801

Luo, Y., Zyu, Y., Luo, D., Zhou, J., Song, E., & Wang, D. (2008). Globally optimal multisensor distributed random parameter matrices Kalman filtering fusion with applications. *Sensors*, *8*, 8086–8103. https://doi.org/10.3390/s8128086

Ma, J., & Sun, S. (2015). Optimal linear estimators for multi-sensor stochastic uncertain systems with packet losses of both sides. *Digital Signal Processing*, *37*, 24–34. https://doi.org/10.1016/j.dsp.2014.11.005

Nakamori, S. (1995). Recursive estimation technique of signal from output measurement data in linear discrete-time systems. *IEICE Trans. Fundamentals of Electronics, Communication and Computer Sciences*, *E78-A* (5), 600–607. https://search.ieice.org/bin/summary.php?id=e78-a_5_600

Nakamori, S. (2019a). Robust RLS Wiener signal estimators for discrete-time stochastic systems with uncertain parameters. *Frontiers in Signal Processing*, *3*(1), 1–18. https://doi.org/10.22606/fsp.2019.31001

Nakamori, S. (2019b). Robust RLS Wiener state estimators in linear discrete-time stochastic systems with uncertain parameters. *Computer Reviews Journal*, *4*, 18–33. https://purkh.com/index.php/tocomp/article/view/355

Nakamori, S. (2019c). Robust RLS Wiener fixed-lag smoother for discrete-time stochastic systems with uncertain parameters. *Computer Reviews Journal*, *4*, 43–57. https://purkh.com/index.php/tocomp/article/view/422

Nakamori, S. (2019d). Robust RLS Wiener fixed-interval smoother in linear discrete-time stochastic systems with uncertain parameters. *Computer Reviews Journal*, *5*, 33–55. https://purkh.com/index.php/tocomp/article/view/552

Qi, W.-J., Zhang, P., & Deng, Z.-L. (2014). Robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances. *Signal Processing*, *99*, 185–200. https://doi.org/10.1016/j.sigpro.2013.12.013

Qi, W.-J., Zhang, P., & Deng, Z.-L. (2014a). Weighted fusion robust steady-state Kalman filters for multisensor system with uncertain noise variances. *Journal of Applied Mathematics*, *2014*, e369252. https://doi.org/10.1155/2014/369252

Qi, W.-J., Zhang, P., & Deng, Z.-L. (2014b). Robust sequential covariance intersection fusion Kalman filtering over multi-agent sensor networks with measurement delays and uncertain noise variances. *Acta Automatica Sinica*, *40*(11), 2632–2642. https://doi.org/10.1016/S1874-1029(14)60410-9

Sun, S.-L. (2004). Multi-sensor optimal information fusion Kalman filters with applications. *Aerospace Science and Technology*, *8*, 57–62. https://doi.org/10.1016/j.ast.2003.08.003

Sun, S.-L., & Deng, Z.-L. (2005). Multi-sensor information fusion Kalman filter weighted by scalars for systems with colored measurement noises. *Journal of Dynamic Systems, Measurement, and Control*, *127*(4), 663–667. https://doi.org/10.1115/1.2101844

Zhang, P., Qi, W.-J., & Deng, Z.-L. (2013). Centralized fusion steady-state robust Kalman filter for uncertain multisensor systems. *Proceedings of 2013 Chinese Intelligent Automation Conference*, 219–226. https://link.springer.com/chapter/10.1007/978-3-642-38460-8_25

Zhang, P., Qi, W.-J., & Deng, Z.-L. (2014). Two-level robust measurement fusion Kalman filter for clustering sensor networks. *Acta Automatica Sinica*, *40*(11), 2585–2594. https://doi.org/10.1016/S1874-1029(14)60404-3

Zhang, R., & Wei, J. Z. (2014). Multi-Sensor Information Fusion and its Application in Robots. *Computer-Aided Design, Manufacturing, Modeling and Simulation III*, *443*, 299–302. https://doi.org/10.4028/www.scientific.net/AMM.443.299.

Conflict of interest

The author declares no conflicts of interest associated with this manuscript.

Author Biography

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