

Fractal properties of the Gas giants and their satellites within the Solar system

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Abstract

This study reveals the fractal structure of gas giants and their moons. For this purpose, fractal analysis of Jupiter, Saturn, Uranus, Neptune and 182 moons was performed based on their radius (size). The results obtained reveal the fractal geometry of the planet / moon systems within the outer Solar system (SS). The resulting fractal dimensions (D) range from -0.57 to -1.43, decreasing with distance from the Sun. This requires a thorough analysis.

Keywords: Solar system, Jupiter, Saturn, Uranus, Neptune, moons, fractal

Introduction

The theory of fractals has been largely developed in the last few decades. The results obtained are frequently used for explanation of the self-similarity and the self-organization of different elements related to the Earth and Planetary science. Thanks to the high achievements of scientific and technical thought in the last half century, mankind has been able to explore the space that has not been available until then. In the course of various space missions, massive data on the geology, topography and physics of the celestial bodies in the solar system has been gathered through space probes and powerful telescopes and satellites. There was a need to develop methods and approaches to analyze and interpret new data. In recent years, fractal analysis has become a major methodological tool for analyzing geological and physical processes and phenomena in others celestial bodies in the solar system [1]. Fractal approach previously has been applied to Mercury's asteroid craters analysis [2], the gravity fields and terrain of Venus ([3]; [4]) and Mars ([3]; [5]; [6]), the topography ([3]; [7]; [8]; [9]; [10]; [11]; [12]) and gravity field peculiarities of the Moon ([13]; [1]; [14]), haze aerosols in Titan's atmosphere [15], and to the Solar system pattern itself [16].

The present study focuses on the fractal properties in the configuration of the four gas giants (outer planets) and their natural satellites (as solid bodies) using their size (radius) (Table 1). The results of the study offer a new interpretation of the principles of structuring of the planetary systems within the Solar system.

Table 1: Radius (size) of the gas giants and their satellites

	JUPITER SYSTEM		SATURN SYSTEM		URANUS SYSTEM		NEPTUNE SYSTEM	
Nº	Name	Mean radius (km)	Name	Mean radius (km)	Name	Mean radius (km)	Name	Mean radius (km)
1	Jupiter	69911	Saturn	58232	Uranus	25362	Neptune	24622

2	Io	2634	Mimas	2576	Ariel	789	Triton	1353
3	Europa	2408	Enceladus	764,5	Umbriel	761	Nereid	170
4	Ganymede	1818	Tethys	735	Titania	585	Naiad	33
5	Callisto	1561	Dione	562,5	Oberon	579	Thalassa	41
6	Amalthea	84	Rhea	536	Miranda	236	Despina	75
7	Himalia	67	Titan	252	Cordelia	81	Galatea	88
8	Elara	49,3	Hyperion	198	Ophelia	75	Larissa	97
9	Pasiphae	43	Iapetus	139	Bianca	68	Proteus	210
10	Sinope	30	Phoebe	107	Cressida	49	Halimede	31
11	Lysithea	23	Janus	90	Desdemona	47	Psamathe	20
12	Carme	21,5	Epimetheus	58	Juliet	40	Sao	22
13	Ananke	19	Helene	47	Portia	40	Laomedea	21
14	Leda	18	Telesto	41	Rosalind	36	Neso	30
15	Thebe	14	Calypso	20	Belinda	32	Hippocamp	17
16	Adrastea	10	Atlas	16	Puck	26		
17	Metis	8,2	Prometheus	16	Caliban	25		
18	Callirrhoe	4,3	Pandora	15	Sycorax	24		
19	Themisto	4	Pan	13	Prospero	21		
20	Megaclite	3,4	Ymir	12	Setebos	20		
21	Taygete	3	Paaliaq	11	Stephano	15		

22	Chaldene	2,7	Tarvos	10	Trinculo	10		
23	Harpalyke	2,6	Ijiraq	9	Francisco	9		
24	Kalyke	2,6	Suttungr	8	Margaret	9		
25	Iocaste	2,5	Kiviuq	8	Ferdinand	6		
26	Erinome	2,2	Mundilfari	6	Perdita	6		
27	Isonoe	2	Albiorix	5	Mab	6		
28	Praxidike	2	Skathi	4	Cupid	6		
29	Autonoe	2	Erriapus	4				
30	Thyone	2	Siarnaq	4				
31	Hermippe	2	Thrymr	4				
32	Aitne	2	Narvi	4				
33	Eurydome	2	Methone	4				
34	Euanthe	2	Pallene	4				
35	Euporie	2	Polydeuces	4				
36	Orthosie	1,9	Daphnis	4				
37	Sponde	1,9	Aegir	3				
38	Kale	1,6	Bebhionn	3				
39	Pasithee	1,5	Bergelmir	3				
40	Hegemone	1,5	Bestla	3				
41	Mneme	1,5	Farbauti	3				
42	Aoede	1,5	Fenrir	3				

43	Thelxinoe	1,5	Fornjot	3				
44	Arche	1,5	Hati	3				
45	Kallichore	1	Hyrrokkin	3				
46	Helike	1	Kari	3				
47	Carpo	1	Loge	3				
48	Eukelade	1	Skoll	3				
49	Cyllene	1	Surtur	3				
50	Kore	1	Anthe	3				
51	Herse	1	Jarnsaxa	3				
52	S/2010 J 1	1	Greip	3				
53	S/2010 J 2	1	Tarqeq	3				
54	Dia	1	Aegaeon	3				
55	S/2016 J 1	1	S/2004 S 7	3				
56	S/2003 J 18	1	S/2004 S 12	2				
57	S/2011 J 2	1	S/2004 S 13	2				
58	Eirene	1	S/2004 S 17	2				
59	Philophrosyne	1	S/2006 S 1	2				
60	S/2017 J 1	1	S/2006 S 3	1				
61	Eupheme	1	S/2007 S 2	1				
62	S/2003 J 19	1	S/2007 S 3	0,3				

63	Valetudo	1	S/2009 S 1	0,15				
64	S/2017 J 2	1						
65	S/2017 J 3	1						
66	Pandia	1						
67	S/2017 J 5	1						
68	S/2017 J 6	1						
69	S/2017 J 7	1						
70	S/2017 J 8	1						
71	S/2017 J 9	1						
72	Ersa	1						
73	S/2011 J 1	1						
74	S/2003 J 2	0,5						
75	S/2003 J 4	0,5						
76	S/2003 J 9	0,5						
77	S/2003 J 10	0,5						
78	S/2003 J 12	0,5						
79	S/2003 J 16	0,5						
80	S/2003 J 23	0,5						

Methods

Number/area approach for fractal analysis

The methodology, based on the correlation number-area, is following the algorithm presented and effectively applied in a previous works dedicated to the solar system ([1]; [16]). The methodological approach uses the following analytical steps:

1. Calculation of total number of study objects (N) with corresponding radius (in respective units) for the graphic.
2. Presentation of the results in graphic form – on the X axis - radii of the study objects are plotted in logarithmic scale, and on the Y axis - the corresponding number in linear scale respectively.
3. The fractal dimension (D) have been calculated using the formula:

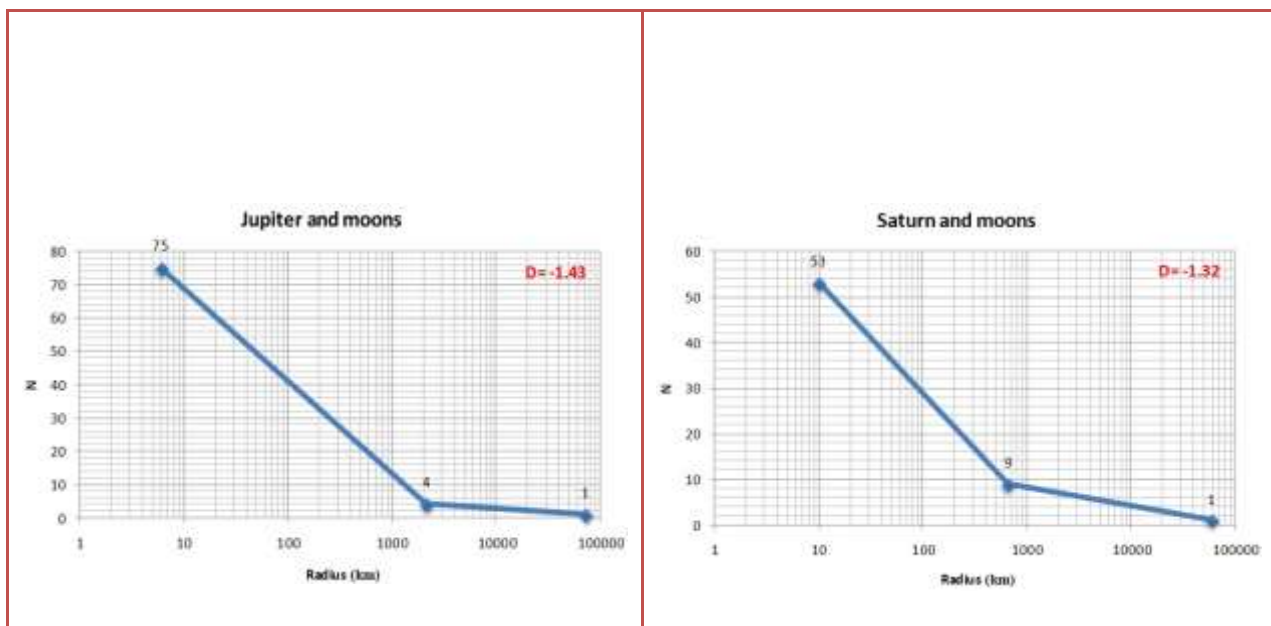
$$D = \Delta \log l / \Delta N \quad (1)$$

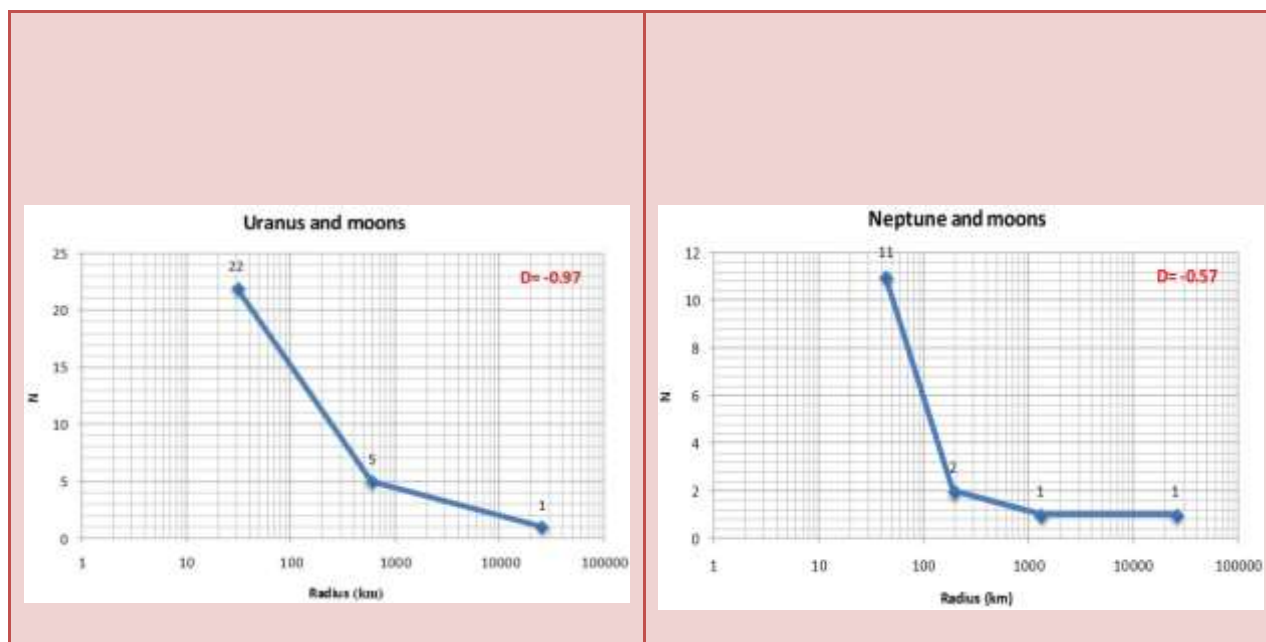
where D is fractal dimension, $\Delta \log l$ is logarithm of iteration and ΔN is total number of objects involved in the study.

Results and Discussion

The results of the fractal analysis of the gas giants and their moons are presented in Table 2. The main conclusions and interpretations related to the results are presented below.

Table 2: Fractal structure of the gas giants and their satellites





The results in Table 2 reveal the fractal structure in the configuration of the gas giants (Jupiter, Saturn, Uranus and Neptune) and their natural satellites. The fractal dimensions range from -0.57 (Neptune) to -1.43 (Jupiter). It is noticeable that the level of fragmentation or fractality decreases progressively with distance from the main center of gravity of our planetary system - the Sun. These results, along with the fractal properties of the Solar system as a whole [16], confirm the fractal configuration of objects within the solar system, both at Solar system and planetary level.

Conclusion

The results on the fractal geometry of the gas giants and their solid satellites offer a new perspective about the evolution of the solar system. The respective gravity forces of each planet configure the respective planet system in relationships with the mass of the Sun and other planets. The satellites (larger or smaller ones) are balanced by the gravity forces to their constant orbits. Formed by the natural forces, all systems represent a fractal structure with corresponding fractal dimension. It is important to mention that the level of fragmentation or fractality decreases progressively with distance from the main center of gravity of our planetary system - the Sun. Future comparative studies would answer whether fractality is a property inherent only in the solar system or is a universal rule occurring everywhere in the universe.

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