World Cup Rugby Japan 2019 the Final, S. Africa vs. England

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Abstract

This paper is concerned with World Cup Rugby Japan 2019 the Final, S. Africa vs. England. A significant improvement of the information dynamic model has been made to make it clear how to solve the boundary layer equation and to know the relation between the flow and game information. For example, the relation between the $1/7^{th}$ velocity distribution law in fluid mechanics and the model in the information dynamics becomes apparent. In the Final, S. Africa always keeps the lead of points throughout the game, but the game itself is quite tight in such a degree that before at 67 minutes within the total time of 80 minutes, England still maintains a chance to reverse the score only by one single try and the following goal. S. Africa keeps the advantage against England, and increases it with increasing the time, and the certainty of game outcome also increases smoothly with increasing the time. The game point of $\eta \approx 0.9$, which is 90% of the total game length(or time), is found as the cross point between the certainty of game outcome and the uncertainty of game outcome, so that the game outcome is firmly definitive at the time between S. Africa's first try and the second try. It is inferred that rugby possesses limitless and uncountable potential to promote people's happiness not only in S. Africa but in any country, and so is worth applying for creating better society in this globe.

Keywords: Society, Information Dynamics, Advantage, Certainty of Game Outcome, Game Point

Introduction

In short, game may be defined as an abstract creature consisting of start, play and end. Including serious game [1] and gamble. It is, therefore, considered that annual ring of a tree, construction of dam or building, soccer, chess, shogi, judo or life of living things are a kind of games, in which methodology for modelling, analyses, prediction of outcome is common in many respects.

Making use of game design patterns, Kelle et al.[2] have implemented information channels to simulate ubiquitous learning support in an authentic situation. Lindley & Sennersten[3]'s schema theory provides a foundation for the analysis of game play patterns created by players during their interaction with a game. Lindley & Sennersten[4] have proposed a framework which is developed not only to explain the structures of game play, but also to provide schema models that may inform design processes and provide detailed criteria for the design patterns of game features for entertainment, pedagogical and therapeutic purposes.

Knowledge about game designs and play patterns has grown fairly well, but little advancement has been made to clarify game history, which denote how information of game varies with the time [5, 6]. Currently the information dynamic model, in which simplicity and generality are characteristics, only makes it possible to treat and identify game history depending on the time. The usefulness of the information dynamic model has been well documented, and successfully applied to baseball [6], effect of medicine [7], soccer [8], shogi [10], or judo [5]. As far as the author is aware of, no existing model is useful, for the applicability is severely limited due to too much sophistication or lack of generality, which prompts the author to develop the novel model [5, 6]. Neumann's game theory [11], for example, provides only a few outcomes due to each the decision by players without considering how they depend on the time: The current game theory merely seeks for the way how to get the maximum gain by keeping the minimum loss in principle.

Main purpose of the present study is to represent the history of the Final, S. Africa vs. England depending on the time precisely by the analyses regarding to the advantage and certainty of game outcome, in terms of information dynamics.

2. Information Dynamic Model

Currently, information dynamic model only make it possible to treat and identify game progress patterns depending on the time. This model has been improved step by step to cope with a wide variety of games since 2012 it appears first [6]. In this model, certainty of game outcome is expressed as a simple mathematical function depending on the time. Simplicity and generality are characteristics of the information dynamic model.

2.1 Modelling Procedure

The modeling procedure of information dynamics based on fluid mechanics is summarized as follows:

(a) Assume a flow problem as the information dynamic model and solve it.

(b) Get the solution(s), depending on the position (or time).

(c) Examine whether any solution of the problem can correspond to game information.

(d) If so, visualize the assumed flow with some means. If not, return to the first step.

(e) Determine the correspondence between the flow solution and game information.

(f) Obtain the mathematical expression of the information dynamic model.

The information dynamic model will be constructed by following the above procedure step by step.

(a) Let us assume flow past a flat plate at incident angle of zero as the information dynamic model

(see Figure 1).

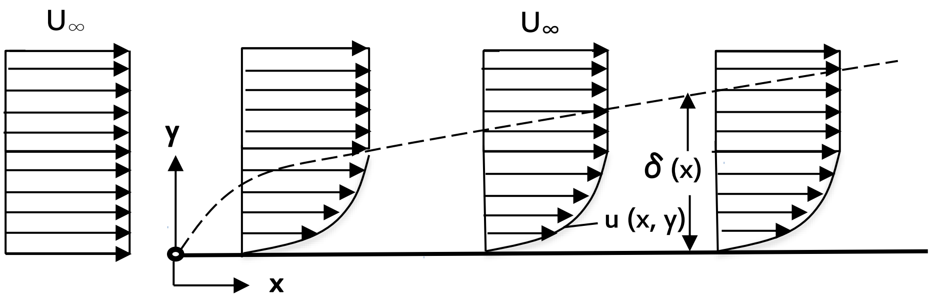

Figure 1 Definition sketch of boundary layer in parallel flow over the flat plate at zero incidence.

An example of the application of the boundary-layer equations, which is the simplified version of Navier-Stokes equations [14], is afforded by the flow along a very thin flat plate at incident angle of zero. Historically, this is the first example illustrating the application of Prandtl's boundary layer theory [13]; it has been analytically solved by Blasius[12] in his doctoral thesis at Göttingen. Let the leading edge of the plate be at x=0, the plate being parallel to the x-axis and infinitely long downstream, as depicted in Figure 1. We shall consider steady flow with a free-stream velocity U_∞, which is parallel to the x-axis. The boundary-layer equations [13, 14] are expressed by

$$u \cdot \partial u/\partial x + v \cdot \partial u/\partial y = -1/\rho \cdot dp/dx + \nu \, \partial^2 u/\partial y^2, \qquad (1)$$

$$\partial u/\partial x + \partial v/\partial y = 0, \qquad (2)$$

$$y = 0 : u = v = 0; \; y = \delta : u = U_\infty, v=0, \qquad (3)$$

where u and v are velocity components in the x- and y- directions, respectively, ρ the density, p the pressure and ν the kinematic viscosity of the fluid. In the free stream,

$$U_\infty \cdot dU_\infty /dx = -1/\rho \cdot dp/dx. \qquad (4)$$

The free-stream velocity, U_∞ is constant in this case, so that dp/dx=0, and obviously dp/dy =0. Since the system under consideration has no preferred length it is reasonable to suppose that the velocity profiles at varying distances from the leading edge are similar to each other, which means that the velocity curves u(y) for varying distances x can be made identical by selecting suitable scale factors for u and y. The scale factors for u and y appear quite naturally as the free-stream velocity, U_∞ and the boundary-layer thickness, δ(x), respectively. Hence the velocity profiles in the boundary-layer can be written as

$u/U_\infty = f(y/\delta)$. (5)

Blasius[12] has obtained an analytical solution in the form of a series expansion around y/δ = 0 and an asymptotic expansion for y/δ being very large, and then the two forms being matched at a suitable value of y/δ.

(b)The similarity of velocity profile is here accounted by assuming that function f depends on y/δ only, and contains no additional free parameter. The function f must vanish at the wall (y = 0) and tend to the value of 1 at the outer edge of the boundary-layer (y = δ), in view of the boundary conditions for $f(y/\delta) = u/U_\infty$.

When using the approximate method, it is expedient to place the point at which this transition occurs at a finite distance from the wall, or in other words, to assume a finite boundary-layer thickness δ(x) in spite of the fact that all exact solutions of boundary-layer equations tend asymptotically to the free-stream associated with the particular problem. The "approximate method" here means that all the procedures are to find approximate solutions to the exact solutions, respectively. When writing down an approximate solution of the present flow, it is necessary to satisfy certain boundary condition for u(y). At least the no-slip condition u = 0 at y = 0 and the condition of the continuity when passing from the boundary-layer profile to the free-stream velocity, so that $u = U_\infty$ at y = δ, must be satisfied.

It may be possible to seek for solution of flow velocity u in the x-direction in (1), by introducing a trial solution (Ansatz Lösung in German) of the form,

$u=ky^m$, (6)

where k is a constant, and m is an exponent of positive real number. Now the present problem becomes to find some suitable values of the exponent m, which may give physically meaningful solutions fulfilling the above boundary conditions [19].

Substituting (6) in (1), we obtain the following solution

$u/U_\infty = (y/\delta)^m$. (7)

Eq.(7) is a series of solutions for the assumed flow as depicted in Figure 1. This represents a series of the solutions with each different value of m. It must be noted that only when m=0, and 1, (7) expresses exact solutions for the assumed flow, but all of the rest solutions are considered to be approximate solutions.

(c) Let us examine whether this solution is game information or not. Such an examination immediately provides us that the non-dimensional velocity varies from 0 to 1 with increasing the non-dimensional vertical distance y/δ in many ways as the non-dimensional information, so that these solutions can be game information.

(d) Imagine that the assumed flow is visualized with neutral buoyant particles. Motion of the visualized particles is detected by the eye almost instantaneously through the lights and is mapped on our retina first [15], so that during these processes, motion of "fluid particles" is transformed into that of the "information particles" by the light carrying the images of fluid particles. This is why motion of the fluid particles is intact in the physical space, but only the reflected lights, or electromagnetic waves consisting of photons can reach at the retina. The photons are then converted to electrochemical particles and are passed along the visual cortex for further processing in parts of the cerebral cortex [15]. The photons and/or electrochemical particles are considered to be information particles. It is, therefore, natural to expect that the flow in the physical space is faithfully transformed to that in the information space, or brain including eye. During this transformation, the flow solution in the physical space changes into the information solution in the information space.

(e) Proposed are correspondences between the flow and game information, which are listed in Table 1.

Table 1 Correspondences between flow and game information

Flow	Game
u: flow velocity	I:current information
U_∞: free stream velocity	I_0: total information
y: vertical co-ordinate	t: current time
δ: boundary layer thickness	t_0: total time

(f) Considering the correspondences in Table 1, (7) can be rewritten as

$$I/I_0 = (t/t_0)^m \qquad (8)$$

Introducing the following normalized variables in (8),

$\xi = I/I_0$ and $\eta = t/t_0$,

we finally obtain the mathematical expression of the information dynamic model as

$$\xi = \eta^m \qquad (9)$$

where ξ is the certainty of game outcome, η is the normalized game length, and m is a positive real number. Simplicity of this model may be noteworthy.

Figure 2 illustrates the relation between certainty of game outcome ξ and normalized game length η, where a total of 10 model curves have been plotted concurrently for reference. This figure clearly indicates generality of this model (9), for each of the curves represents a game. Thus, this model can represent any game in principle, where each of games takes a unique value of m, number of which is limitless. The smaller the strength difference between both the teams (or players) is, the greater the value of m, and *vice versa*.

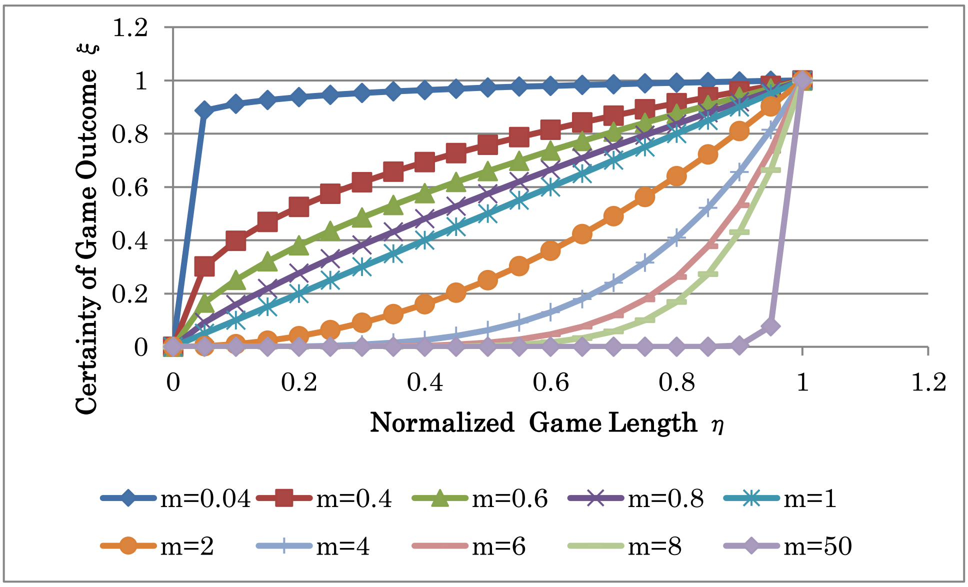

Figure 2 Certainty of game outcome ξ against normalized time η

It may be instructive to rewrite (9) to relate with the previous known works [16, 17, 18] in fluid mechanics, as

$$\xi = \eta^{1/n}, \quad (10)$$

where m=1/n. Then, this time, let us draw (10) in such a way that the normalized flow velocity ξ is abscissa, while the normalized boundary layer thickness η is ordinate, as shown in Figure 3. It is certain that each of the curves in this figure represents the velocity profile in the boundary layer depicted in Figure 1. In particular, the straight line at n=1(m=1) in Figure 3 shows an exact solution in (1).

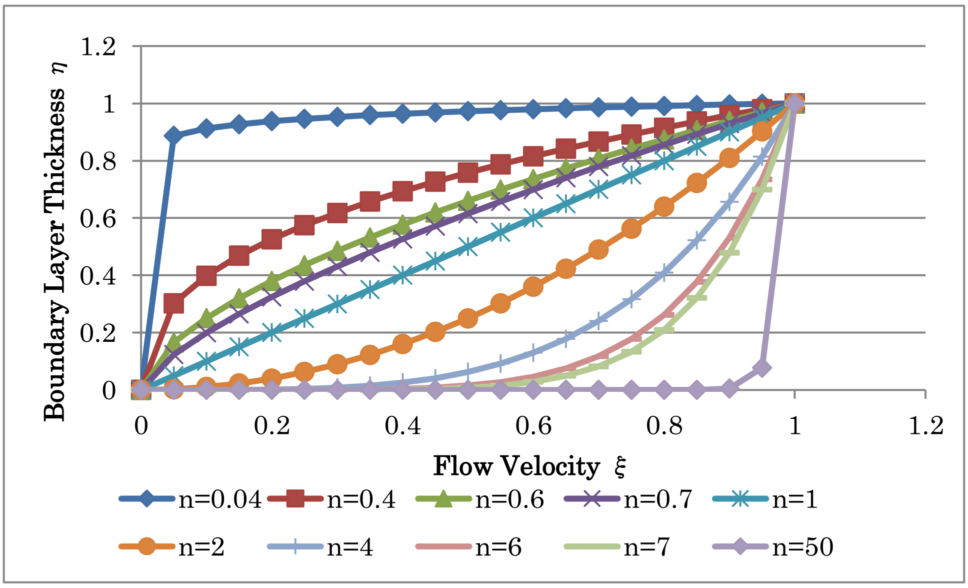

Figure 3 Normalized flow velocity ξ against normalized boundary layer thickness η.

Nikuradse[16] carried out a very thorough experimental investigation into the law of friction and velocity profiles in smooth pipes in a very wide range of Reynolds numbers $4x10^3 \leq Re \leq 3.2x10^6$. They are given in dimensionless form in that u/U_∞ has been plotted against y/R. It is realized that the velocity profile becomes fuller as the Reynolds number increases. It is possible to represent it by the empirical equation, known as the power velocity distribution law, viz.

$$u/U_\infty = (y/R)^{1/n}, \quad (11)$$

where the exponent n varies with the Reynolds number. Similarity of (11) to (10) is notable, though the former is the empirical, but the latter is theoretical. On one hand, the power velocity distribution law has been derived from Blasius's resistance formula by Wieghardt[17], as

$$u/u_* = C(n)(yu_*/\nu)^{1/n}, \quad (12)$$

where C(n) is constant depending on the exponent n, $u_* = (\tau/\rho)^{1/2}$ denotes the friction velocity, τ is the shearing stress at the wall, and ρ is the density of the fluid. Wieghardt gives us the set of the exponent n and the constant C(n), as listed in Table 2

Table 2 The relation between the exponent n and the constant C(n) in the power velocity distribution law given by Wieghard[17].

n:	7,	8,	9,	10
C(n):	8.74	9.71	10.6	11.5

Paeschke[18] has proposed another power velocity distribution law as

$$u/u_1 = (y/y_1)^{1/n}, \quad (12)$$

where u_1 is the measured velocity, which is at the distance y_1 from the wall. It has been verified that this law is reliable for the flow over the smooth and rough walls, respectively, and for the flow in high free atmosphere. Note that the exponent n depends not only on height, but also on the roughness of the wall. The value of the exponent n in the flow over the flat plate and/or in the pipe is 7 or 8 as suggested by Prandtl [20] in 1929, but in the free atmosphere this value is normally between 2 and 5, as listed in Table 3.

Table 3 Dependency of the exponent n on the surface roughness and the condition.

	Vegetable field	Grain field	Grass field	Waste land	Airport	Snow field
n:	3.0	3.5	3.6(3.85)	4.0	4.3	5.0

In short, the value of exponent n in (10) varies with the Reynold number, roughness on the wall, height, temperature, and deceleration and/or acceleration of flow. How these factors affect to the game dynamics is a promising research topics, to be explored by the researchers in fluid mechanics and in information science. For the approximate solutions in the boundary layer flow over a flat plate at zero incidence, see Appendix.

2.2 Method of Analyses

Elemental procedure for obtaining the advantage α, certainty of game outcome ξ, and uncertainty of game outcome ς will be explained by using soccer game between teams A and B, where only goal(s) is treated as the evaluation function score for clarity.

The advantage α is defined as follows: When the total scores of the two teams at the end of game $S_T \neq 0$,

$\alpha = [S_A(\eta) - S_B(\eta)]/S_T$ for $0 \leq \eta \leq 1$, (13)

where $S_A(\eta)$ is the current goal sum for team A(winner), $S_B(\eta)$ is the current goal sum for team B(loser), and η is the normalized game length, which is normalized by the total time.

When α>0, team A (winner) gets the advantage against team B (loser) in the game, while when α<0, team B (loser) gets the advantage against team A (winner). It is certain that when α=0 the game is balanced. When the total scores of the two teams at the end of game $S_T = 0$, α=0 for $0 \leq \eta \leq 1$

The certainty of game outcome means what extent the game outcome (i.e., win or loss) is certain depending on the time during the game, and the extent is given by the normalized value ranging from 0 to 1. The certainty of game outcome ξ during the game is defined as follows: When the total scores of the two teams at the end of game $S_T \neq 0$,

$\xi = |S_A(\eta) - S_B(\eta)|/S_T$ for $0 \leq \eta < 1$

=1(completed game) for η=1 or

= 0(drawn game) for η=1. (14)

At η=1, ξ is assigned to the value of 1, for at the end of completed game the information on the game outcome must be full certainty. Note that ξ takes always positive value, for it is the absolute value of advantage α. The reason why we take the absolute value of advantage α to get certainty of game outcome ξ for $0 \leq \eta < 1$ is that certainty of game outcome ξ is independent of the sign of advantage α: As the absolute value of advantage α increases (decreases), certainty of game outcome ξ must increase (decrease). In case of drawn game, certainty of game outcome ξ may be assigned to the value of 0 at the end of game η=1, for the game is right back where it starts. When the total scores of the two teams at the end of game $S_T = 0$, ξ=0 for $0 \leq \eta \leq 1$.

The uncertainty of game outcome ς during the game is defined as follows

$$ς = 1 - ξ. \quad (15)$$

Keeping in mind the forgoing elemental procedure to obtain the advantage α, and certainty of game outcome ξ, it is straight forward to apply them to actual rugby games. In rugby, it may be evident that points by try, goal and/or penalty goal are evaluation function scores, for it is a critical factor for the game. In computer shogi, evaluation function score for each of the piece move has been assessed by a human similarly to that in computer chess based on objective knowledge and experiences [10]. Since accuracy of the evaluation function score for each of the piece move has been improved by repeating trial and error in such a level that Bonkras(Computer Shogi World Champion in 2012) beats Kunio Yonenaga(Shogi Champion or *Meijin*). Today, evaluation function score for each of the piece move in popular board games such as chess, shogi and/or go can be calculated by using computer soft quite accurately.

3. Case Study

In this section, World Cup Rugby Japan 2019, the Final, S. Africa vs. England, held at Yokohama International Stadium on Nov. 2, 2019 will be analyzed and discussed.

3.1 Prediction of the Final

The World Rank of S. Africa is no.4, while that of England is no.2 before the Final. S. Africa beats Japan by the score of 26:3 in the final tournament, and beats Wales by the score of 19:16 in the Semi-final, whereas England beats Australia by the score of 40:16 in the final tournament, and beats New Zealand by the score of 19:7 in the Semi-final, and New Zealand is no.1 in the World rank. As far as considering the above records, England seems to keep the advantage against S. Africa. Actually, majority of the news or reports estimates that England will beat S. Africa, even though the game itself must be severely balanced, and thus the game may get entangle, and the outcome cannot be identified until near the end.

3.2 Brief history of the Final, S. Africa vs. England :

Throughout the game, S. Africa overwhelms England in both of offense and defense and set play as well. The preemptive penalty goal by the Pollard succeeds at 9 minutes after the kick off. At the England field, S. Africa pushes England by the scrum strongly. Repeating the scrums, S. Africa gets the penalty goals at 25, 38, 42,47, and 59 minutes, respectively, though England gets the penalty goals only at 22, 34, 53, and 61 minutes, respectively. Then, S, Africa makes the decisive try at 67 minutes, and the doubly sure try at 75 minutes. The history of points taken during the game is summarized in Table 4.

The S. Africa's style depending on the physical strength of players represents the pride based on their culture and history. Contrary to New Zealand's style depending on the quick and elegant ball pass, S. Africa sticks to the physical attack to the opponents during the offense and defense.

Since abolition of black race isolation policy, as symbolized by the historical victory of S. Africa in World Cup Rugby S. Africa 1995, rugby has the special meaning for the people in S. Africa, though the outcome is not always satisfactory and their expectation has been often betrayed. However, the declined situation has changed dramatically once Elasms succeeds the position of supervisor: He made a speech on the special value of rugby for the people in S. Africa at the inaugural meeting of the national team. The first negro captain, Colisi reflected that meeting in such a way " I have changed my mind due to Erasms' speech. I made my mind to do hard work so as to pursue only one thing, victory for our people, as much as possible. I realized this is most important for our team".

After the Final, supervisor, Erasms said " we get the champion cup, so that I am very pleased to obtain it, and at the same time I am proud of this honor without giving up the long persisting dream, though this result may be contrary to the expectation by the majority relating to Rugby fans, we regain confidence and honor by arriving at the peak. In our country, there are many social problems still, but we will do our best to let our people happy. The captain, Colisi said "gathering players originating in various back grounds and races, we become one single team ultimately to get this title". The kicker, Pollard made another comment "since I was a small boy, I have trained penalty goals of the World Cup in the small back garden. Tonight, the dream comes true". Pollard becomes the king of score by taking points of 69 during this World Cup.

Table 4 History of points taken during the Final, S. Africa vs. England

Time [min.]	S. Africa	England	
0	0	0	
9	3	0	PG
22	3	3	PG
25	6	3	PG
34	6	6	PG

38	9	6	PG
42	12	6	PG
47	15	6	PG
53	15	9	PG
59	18	9	PG
61	18	12	PG
67	25	12	T, G
75	32	12	T, G
80	32	12	

Table 4 indicates the way how S. Africa beats to England depending on the time by the score of 32 to 12. That is, S. Africa always keeps the lead of points throughout the game. But, actually, this game is rather tight one, for until S. Africa gets the first try at 67 minutes, the point difference between S. Africa and England is within 6 point except for 9 points at 59 minutes. This means that before at 67 minutes England has the chance to reverse the score only by one single try and the following goal. Table 5 may support this view, for the ball control rate of England even exceeds to that of S. Africa by 12%, though S. Africa is superior to England in tackle success rate and line out success rate.

Table 5 Miscellaneous game information

	S. Africa	England
Ball control rate	44%	56%
Tackle success rate	92%	89%
Scrum success rate	100%	100%
Line out success	100%	88%
Number of foul	8	10

3.2 Results of Data Analyses

In this section, how the advantage and certainty of game outcome depend on the normalized game length (or time) will be presented by the data analyses.

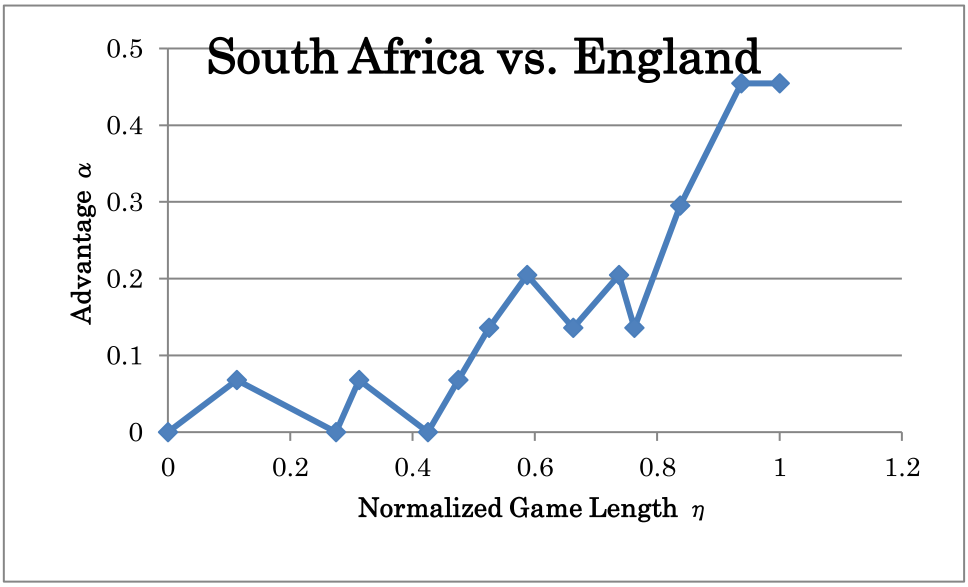

Figure 4 Advantage α against normalized game length η

Figure 4 shows the relation between the advantage α and the normalized game length η, where the abscissa is normalized by the total time of 80 minutes, while the ordinate is normalized by the total score of 44 points. This figure shows that S. Africa always keeps the advantage against England, and increases it with increasing the normalized game length, though the advantage takes the value of 0 twice.

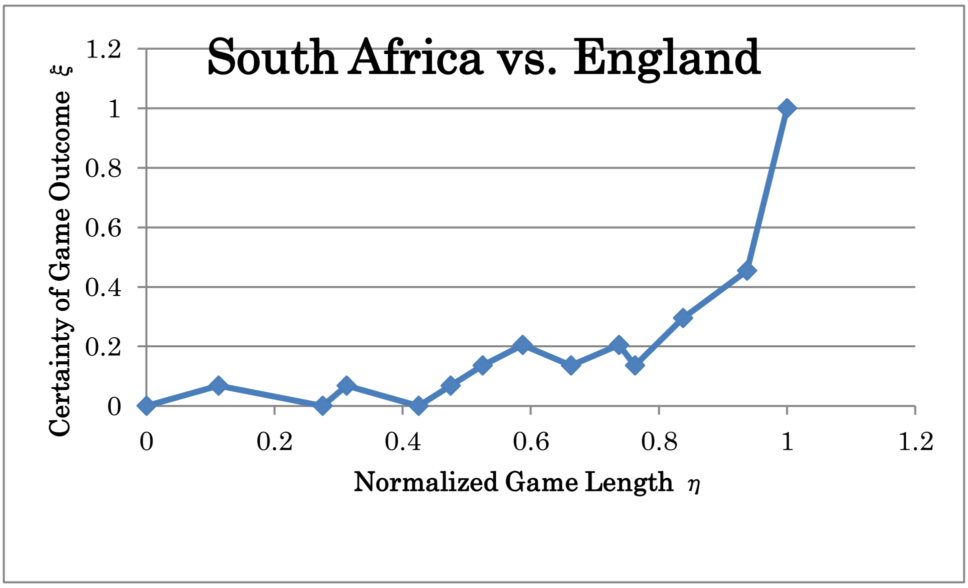

Figure 5 Certainty of game outcome ξ against normalized game length η

Figure 5 shows the relation between the certainty of game outcome ξ and the normalized game length η. This figure shows that the certainty of game outcome ξ increases smoothly with increasing the normalized game length η, though at two points ξ takes the value of 0.

4. Discussion

In this, section, the game point, at which the game outcome becomes definitive, will be obtained in terms of the information dynamic models. That is, once the game length (or time) exceeds to the value of this point, the result of the game cannot be altered by any means, or the game outcome is fixed. This substantial notion in game is introduced by Nakagawa & Minatoya[5] for the first time, and is coined as 'game point'.

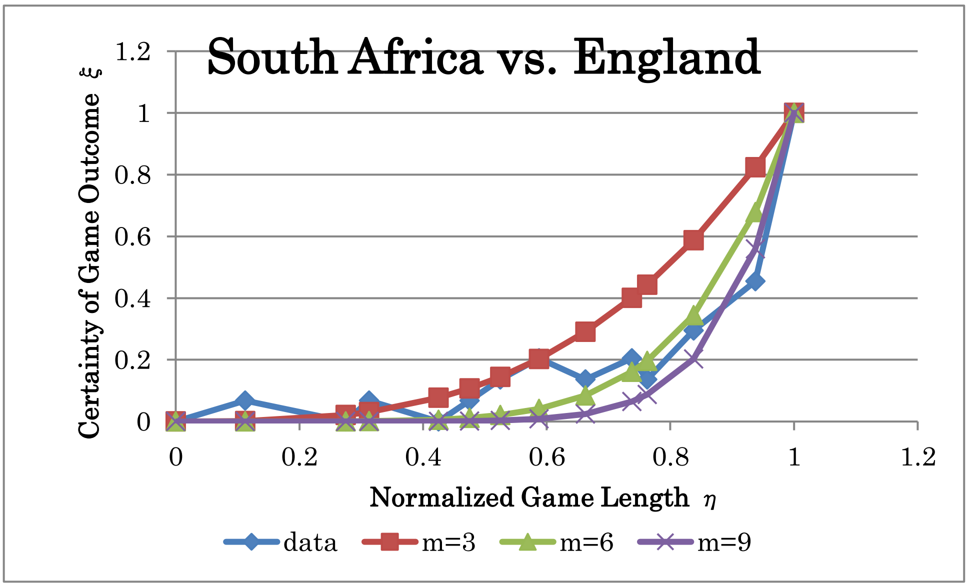

Figure 6 Certainty of game outcome ξ against normalized game length η

Firstly, the best fit model curve to the data curve will be sought by plotting the data curve and model curves concurrently, and then using the best fit curve, the game point will be found by knowing the cross point of the certainty of game outcome and the uncertainty of game outcome.

In Figure 6, the data curve of certainty of game outcome and the three model curves $\xi=\eta^3$, η^6, and η^9 are plotted concurrently. It may be evident that the model curve $\xi=\eta^6$ fits to the data curve best.

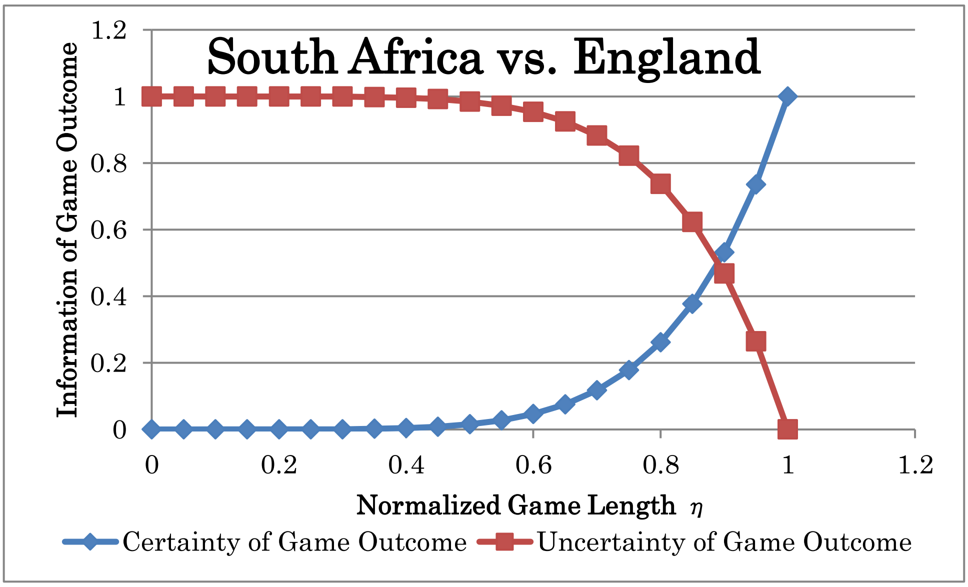

Figure 7 Information of game outcome against normalized game length η.

Figure 7 shows the concurrent plot of the certainty of game outcome ξ and the uncertainty of game outcome ς to find the game point, which is the cross point between these two curves: It is found that the game point is at $\eta \approx 0.9$, so that it is considered that the game outcome becomes definitive at this point. Referring to Figure 5, the game length (or time) of the game point corresponds to that between S. Africa' s first try and the second try. In another words, the game outcome of the Final S. Africa vs. England is not certain even after S. Africa gets the first try, and so take the score lead by 13 points. Actually, the game outcome is fixed firmly at the moment of S. Africa's second try.

5. Conclusions

In this section, new knowledge and insights obtained through the present study have been summarized.

1. A significant improvement of the information dynamic model has been made to make it clear by solving the boundary layer equation and to know the relation between the flow and game information. For example, the relation between the $1/7^{th}$ velocity distribution law in fluid mechanics and the model curve in the information dynamics becomes apparent.
2. In the Final, S. Africa always keeps the lead of points throughout the game, but the game itself is quite tight in such a degree that before at 67 minutes within the total time of 80 minutes, England still maintains a chance to reverse the score only by one single try and the following goal.
3. S. Africa keeps the advantage against England, and increases it with increasing the time, and the certainty of game outcome increases smoothly with increasing the time.

4. The game point of η≈0.9, which corresponds to 90% of the total time, is found as the cross point between the certainty of game outcome and the uncertainty of game outcome, so that the game outcome becomes definitive at the time between S. Africa's first try and the second try.
5. It is inferred that Rugby possesses limitless and uncountable potential to promote people's happiness not only in S. Africa but in any country, and so is worth applying for creating better society in this globe.

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Appendix: Approximation solutions in the boundary layer flow +

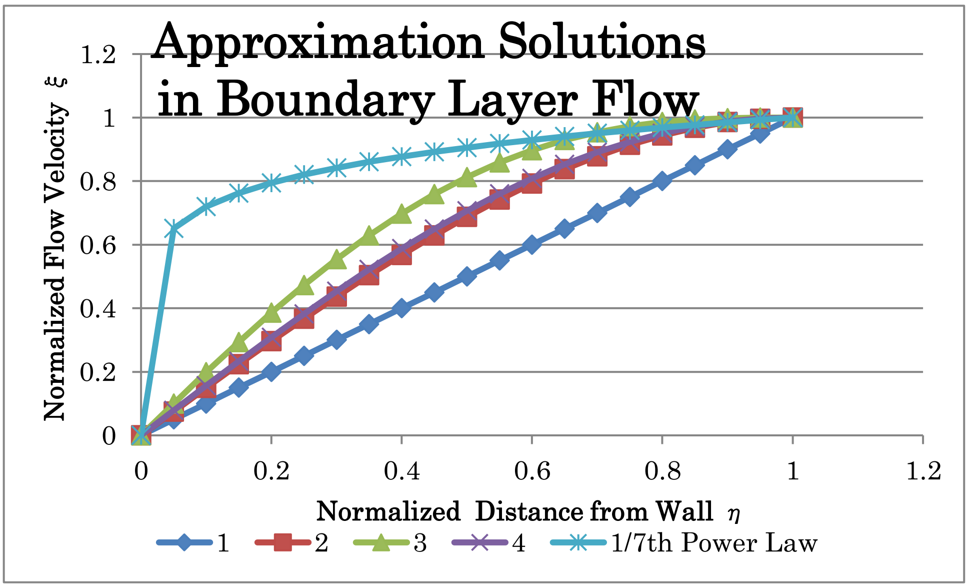

Figure 8 Normalized flow velocity in boundary layer ξ against normalized distance from the wall

Approximate solutions in the boundary layer flow over a flat plate at zero incidence have been plotted in Figure 8, where the abscissa is the normalized distance η from the wall, whereas the ordinate is the normalized velocity ξ.

The mathematical expression of these approximate solutions are as follows,

Curve 1: $\xi=\eta$,

Curve 2: $\xi=3/2*\eta-1/2*\eta^3$,

Curve 3: $\xi=2*\eta-2*\eta^3+\eta^4$,

Curve 4* $\xi=\sin(\pi/2*\eta)$,

1/7th Power Law: $\xi=\eta^{1/7}$.

It is well known that the 1/7th power velocity distribution law is most realistic one [18, 20], but the rest velocity distribution curves 1, 2, 3, and 4 are different from it, respectively, as shown in Figure 8.

The curve 1 satisfies only the conditions $\xi(0)=0$ and $\xi(1)=1$, whereas curve 2 satisfies in addition to these two conditions, $d\xi/d\eta(1)=0$ and $d^2\xi/d\eta^2(0)=0$; finally, curve 3 can be made to satisfy the additional condition $d^2\xi/d\eta^2(1)=0$. The curve 4, sine function satisfies the same boundary conditions as the polynomial of fourth degree, the curve 3, except for $d^2\xi/d\eta^2(1)=0$. The polynomials of third and fourth degree and the sine-function lead to values of shearing stress at the wall which are in error less than 3 %, and thus may be considered entirely adequate to this respect. Furthermore, the values of the displacement thickness δ_1 for the curves 1, 2, 3, and 4, respectively show acceptable agreement with the corresponding exact value due to Blusius[12], where the displacement thickness is defined by

$$\delta_1 = \int_{□=0}^{\infty} (U_\infty - u)dy/U_\infty.$$

It is now evident that the approximate solutions provide satisfactory results in the case of the parallel flow over a flat plate at zero incidence, and the extraordinary simplicity of the calculation is quite remarkable, compared with the complexity of the exact solution due to Blasius[12].